Some higher-dimensional analogues

M complete Riemannian mfd of dim. n

Myers Theorem:

$\mathrm{Ric} \geq 0$ & M compact $\Rightarrow$ $\begin{cases} \mathrm{Vol}(M) \text{ bded} \\ \pi_1(M) \text{ finite} \end{cases}$

(believed that $\pi_1(M)$ must be cyclic).

Theorem: $\mathrm{Ric} \geq 0$ & M compact $\Rightarrow$ $\pi_1(M)$ has polynomial growth of deg $\leq n$.

Theorem (Milnor '63): $\mathrm{Ric} \geq 0 \Rightarrow$ every finitely generated subgp of $\pi_1(M)$ has polynomial growth of deg $\leq n$ (with different constants).

Milnor's Conjecture:

$\mathrm{Ric} \geq 0 \Rightarrow \pi_1(M)$ finitely generated.

Example of non-complete surface:

$S^2 - C$ (C Cantor set).