Applications of Macdonald Polynomials

F. Bergeron (Université du Quebec a Montréal),
J. Haglund (University of Pennsylvania),
J. Remmel (University of California at San Diego)

9/9/2007 – 9/14/2007

1 Overview of the Field

In 1988 Macdonald [33], [34] introduced a new family of symmetric functions $P_\lambda(X; q, t)$ depending upon a partition λ, a set of variables $X = \{x_1, \ldots, x_n\}$, and two real parameters q, t. They were immediately hailed as a breakthrough in symmetric function theory as well as special functions, as they contained most of the previously studied families of symmetric functions as special cases, and yet satisfied many exciting properties, such as a multivariate orthogonality relation. Some of these properties were conjectural, like Macdonald's postivity conjecture for the coefficients $K_{\lambda,\mu}(q, t)$ in the expansion of P_λ into the "plethystic Schur" basis $s_\mu[X(1-t)]$, which became a famous problem. Garsia and Haiman [10] refined this conjecture, giving a representation theoretic interpretation for the coefficients in terms of Garsia-Haiman modules, an interpretation which was finally proved ten years later in 2000 by Haiman, who connected the problem to the study of the Hilbert scheme of n points in the plane from algebraic geometry [17]. Another famous problem, Macdonald's constant term conjectures, involved an extension of the P_λ to arbitrary affine root systems. (In this setting, λ is no longer a partition, but an element of a certain lattice associated to the root system.) Letting $<, >_{q,t}$ denote Macdonald's scalar product with resect to which the P_λ are orthogonal, which can be expressed as the constant term in a certain multivariate Laurent series, Macdonald introduced a specific value for $< P_\lambda, P_\lambda >_{q,t}$ which in type A reduced to the q-Dyson conjecture. After several special cases were proved by a varity of authors, Macdonald's constant term conjectures in full generality were finally solved in the mid 1990's by Cherednik [4], [5], who showed they have a natural interpretation in terms of the representation theory of fundamental objects he introduced now called double affine Hecke algebras, or Cherednik algebras.

In 1995 Macdonald introduced [35] polynomials $E_\alpha(X; q, t)$, where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is a weak composition into n parts. He showed these polynomials also satisfy an orthogonality relation, and are a basis for the polynomial ring $\mathbb{Q}[x_1, \ldots, x_n](q, t)$. The E_α can be thought of as a refinement of the P_λ, since the main properties of the P_λ can be easily derived from corresponding properties of the E's. Cherednik developed the theory further [6], showing how the E_α also have an interpretation in terms of the representation theory of the double affine Hecke algebra.

In conjunction with their study of the study of the $K_{\lambda,\mu}(q, t)$, Garsia and Haiman [16], [11] introduced a number of fascinating combinatorial problems involving the space of diagonal harmonics DH_n, an important S_n module which contains the Garsia-Haiman modules as S_n sub-modules. (DH_n is isomorphic to the quotient ring of diagonal coinvariants.) For example, Haiman conjectured the dimension of DH_n is $(n + 1)^{n-1}$, which equals the number of parking functions on n cars. He also conjectured the dimension of the subspace of diagonal harmonic alternants DH_n^ϵ is the nth Catalan number C_n. By taking into account the natural bigrading of these spaces we get q, t versions of the number of parking functions and also the nth Catalan number. Based on some ideas of Procesi, Garsia and Haiman were led to a conjectured formula for the character of DH_n, which involved a sum of rational functions, with symmetric functions $\tilde{H}_\mu(X; q, t)$, modified versions of the Macdonald polynomials, occurring in the numerators. The terms in this formula correspond to terms in the Atiyah-Bott-Leftschetz fixed point formula, which is connected to the problem via an underlying torus action on the Hilbert scheme. In 2000 Haglund [14] introduced a specific conjectured interpretation for the q, t-Catalan number, which was proved shortly after in joint work with Garsia [8], [9]. The proof used plethystic symmetric function identities involving Macdonald polynomials which were developed by F. Bergeron, Garsia, Haiman, and Tesler during the 1990's [2], [12]. In 2001 Haiman proved the rational function formula for the character of DH_n using algebraic geometry[17]. Three years later Haglund, Haiman, Loehr, Remmel and Ulyanov [21] introduced a conjecture which is still open, the "shuffle conjecture" which gives a a combinatorial conjecture for this character in terms of statistics on parking functions. Haglund's book [15] includes a detailed discussion of the combinatorics of DH_n, while Bergeron's book [1] contains further information on DH_n in the context of a more general discussion on coinvariant spaces. In another direction Iain Gordon [13] proved a conjecture of Haiman for the dimension of versions of DH_n for other Weyl groups. A refinement of this result, involving an extra parameter q was later found by Cherednik [7].

2 Recent Developments and Open Problems

Although from the time of their introduction Macdonald polynomials enjoyed a very fruitful interaction between harmonic analysis and representation theory, until fairly recently little was known about the combinatorics. In 2004 Haglund introduced a conjectured combinatorial model for the expansion of the $\tilde{H}_\mu(X; q, t)$ into monomials, or into Gessel's fundamental quasisymmetric functions. The conjecture was proved shortly after by Haglund, Haiman, and Loehr, who also showed the formula led easily to an expansion of $\tilde{H}_\mu$ into LLT polynomials, symmetric functions, depending on a parameter q, introduced by Lascoux, Leclerc, and Thibon. These functions were conjectured to be Schur positive, and now Grownowski and Haiman have announced a proof of this conjecture, which gives a new proof (using the representation theory of Hecke algebras) of Macdonald's positivity conjecture that the $\tilde{H}_\mu$ are Schur positive. Moreover, Sami Assaf has developed an amazing combinatorial model which algorithmically constructs a graph from each LLT polynomial, which also proves Schur positivity of the LLT by giving an explicit recursive construction for the Schur coefficients. The details of Assaf's construction are rather complicated though, and a major open question is whether her construction can be simplified to obtain nice formulas for the $K_{\lambda,\mu}(q, t)$.

Another significant body of recent research with implications to Macdonald theory is work on the k-Schur function. This is a family of symmetric functions in a set of variables X which depends on a partition λ, a positive integer k, and a parameter q, and which reduces to the usual Schur function when k is large enough. Originally introduced by Lascoux, Lapointe, and Morse [25], the combintorial theory of the k -Schur was primarily developed by Lapointe and Morse during the period $2000 - 2007$ [26], [27], [28], at which time they discovered connections between the k-Schur and Gromov-Witten invariants [29]. This led a number of other researchers such as Lam, Shimozono, and Schilling to work on the subject. It is conjectured that when LLT polynomials of total "band width" k are expressed in terms of the k-Schur, the coefficients are nonnegative polynomials in q. This implies Macdonald's positivity conjecture. There are currently about seven different ways of defining the k-Schur, which are all conjectured to be equivalent, and there is also a growing body of conjectures surrounding the combinatorial properties of the k-Schur. Billey and Assaf have recently announced a solution to one of these conjectures, that one of the definitions of the k-Schur is Schur-positive, by utilizing Assaf's LLT-graph decomposition algorithm.

There have been a number of interesting recent developments in the study of the combinatorics of DH_n. First of all, Loehr and Warrington have introduced a very general conjecture of the following form: let ∇ be a linear operator defined on the $\tilde{H}_\mu(X; q, t)$ by

$$\nabla \tilde{H}_\mu(X; q, t) = t^{n(\mu)} q^{n(\mu')} \tilde{H}_\mu(X; q, t), \tag{1}$$

where $n(\mu) = \sum_i (i-1)\mu_i$. F. Bergeron first noticed that many important identities in Macdonald theory can be elegantly expressed using the∇ operator. Loehr and Warrington [32] give an explicit combinatorial expression for ∇ applied to any Schur function s_λ, as a sum over statistics on parking functions for "nested" lattice paths. If $\lambda = 1^n$, the Loehr-Warrington conjecture reduces to the shuffle conjecture. Another generalization of the shuflfle conjecture has been introduced by Haglund, Morse, and Zabrocki. By building on earlier work of N. Bergeron, Descouens, and Zabrock [3], they conjecture that ∇ applied to a Hall-Littlewood function can be expressed in terms of statistics on parking functions for lattice paths which hit the main diagonal in certain specified points. Garsia, Xin, and Zabrocki have announced a proof of the hook shape case of this conjecture, which generalizes Haglund's q, t-Schröder theorem. Both the Loehr-Warrington conjecture and the Haglund-Morse-Zabrocki conjecture give expressions for ∇ applied to a whole basis for the ring of symmetric functions, which hopefully will be easier to prove than looking at ∇s_{1^n} by itself.

In [19] Haglund, Haiman, and Loehr give a version of the combinatorial formula for the $\tilde{H}_\mu(X; q, t)$ for the $J_\lambda(X; q, t)$ (scalar multiples of the P_λ whose monomial coefficients are in $\mathbb{Z}[q, t]$), and in subsequent work [20] also a version involving the $\mathcal{E}_\alpha(X; q, t)$ (scalar multiples of the $E_\alpha(X; q, t)$ whose monomial coefficients also have no denominators). The formula for the $\mathcal{E}_\alpha(X; q, t)$ involves a sum over certain "nonattacking" fillings, of the diagram whose $n - i + 1$st column has height α_i, with positive integers, with each such filling weighted by powers of q, t and also factors of the form $(1 - q^a t^b)$ for certain powers a, b defined combinatorially. This model also contains a "basement", i.e. a row of squares below the diagram filled with the numbers $n, n-1, \ldots, 2, 1$. By changing the basement to $2n, 2n-1, \ldots, n+1$ and summing as before over nonattacking fillings we get a formula for J_μ, where μ is the rearrrangement of the parts of α into partition order. Thus there are actually a number of formulas for J_μ corresponding to the various ways to permute the parts of μ and shuffle with zeros to obtain a weak composition.

Ram and Yip [39] have introduced a general formula for the E_α for arbitrary affine root systems. Their formula is obtained by iterating recurrence relations which can be used to define the E_α (known as intertwiner relations), and is expressed in terms of "alcove walks" in a certain lattice associated to the root system. In type A their formula has many more terms than the formula in [19], but Lenart [31] has shown how to group together terms in the J_μ version of their formula to obtain exactly the formula from [20] for J_μ corresponding to the case where $\alpha = \mu$. An exciting question for future research is whether or not terms in the Ram-Yip formula for general affine root systems can be grouped together in a similar way to obtain a canonincal combinatorial formula for the E_α.

Since both Demazure characters (also known as key polynomials), and the standard bases (introduced by Lascoux and Schützenberger [30] in their study of Schubert varieties) are limiting or special cases of the E_α, new combinatorial formulas for these functions are a by-product of the new Macdonald combinatorics. (See [40] for more background on key polynomials.) In connection with her study of these identities Sarah Mason [37], [38] introduced a generalization of the RSK algorithm. Recently Haglund, Luoto, Mason, and van Willigenburg have introduced a new basis $\mathrm{QS}_\beta(X\}$ for the ring of quasisymmetric functions they call quasisymmetric Schur functions, and have used properties of Mason's RSK algorithm to prove the QS_β satisfy a generalzation of the Littlewood-Richardson rule [22], [23]. Lauve and Mason have announced they have been able to use this generalized Littlewood-Richardson rule to prove a conjecture of F. Bergeron and C. Reutenauer that gives an explicit basis for the quotient ring of quasisymmetric functions in n variables by the ring of symmetric functions in n variables.

One implication of the type A formula for J_μ in [19] is that the coefficient of a monomial symmetric function in

$$J_\mu(X; q, q^k)/(1-q)^n \tag{2}$$

is in $\mathbb{N}[q]$, for any positive integer k. Maple calculations led Haglund to conjecture the stronger relation that the coefficient of a Schur function in (2) is in $\mathbb{N}[q]$. Ram has suggested that a more general phenomena may hold, where you decompose $J_\mu(X; q, q^k)$ in terms of the basis $\{J_\lambda(X; q, q^{k-1})\}$, with some kind of positivity at each step ($J_\lambda(X; q, q)$ is a scalar multiple of the Schur function $s_\lambda(X)$).

3 Presentation Highlights

Many of the talks at the workshop, for example talks by N. Bergeron, I. Gordon, J. Haglund, N. Loehr, S. Mason, and J. Morse, involved topics discussed in the above two sections. Other talks were about other topics relevant to symmetric function theory and Macdonald polynomials of interest to researchers in this area. Below we include titles and abstracts for all the presentations.

Abstracts for Talks

Speaker: **Nick Loehr** (Virginia Tech, USA) (talk describes joint work with Jim Haglund and Mark Haiman)
Title: *Symmetric and Non-symmetric Macdonald Polynomials*
Abstract: Macdonald polynomials have played a central role in symmetric function theory ever since their introduction by Ian Macdonald in 1988. The original algebraic definitions of these polynomials are very non-explicit and difficult to work with. Haglund conjectured an explicit combinatorial formula for the Macdonald polynomials. This was later extended to a combinatorial formula for non-symmetric Macdonald polynomials in type A. This talk will discuss the algebraic and combinatorial definitions of both symmetric and non-symmetric Macdonald polynomials. We also sketch the main ideas in the proofs that the algebraic and combinatorial constructions are equal.

Speaker: **Jim Haglund** (Univ. of Pennsylvania, USA) will deliver a talk prepared by **Greg Warrington** (Wake Forest, USA) who had to cancel his trip
Title: *Combinatorical structures associated to the nabla operator*
Abstract: Over the past ten years, there has been a rich interplay among the modified Macdonald polynomials, the diagonal harmonics modules, the nabla operator, and the combinatorics of q,t-weighted lattice paths. In this talk, we review these connections, paying particular attention to the q,t-Catalan numbers. We finish with recent joint work of N. Loehr and G. Warrington regarding a nested-lattice-path interpretation for nabla applied to arbitrary Schur functions.

Speaker: **Sami Assaf** (Univ. of Pennsylvania, USA)
Title: *A combinatorial proof of Macdonald positivity*
Abstract:Taking Haglund's formula for the transformed Macdonald polynomials expressed in terms of monomials as the definition, we present a self-contained, combinatorial proof of symmetry and Schur positivity of Macdonald polynomials, and give a combinatorial interpretation of the Schur coefficients. The method of the proof uses the theory of dual equivalence graphs and a new generalization of them called D graphs.

Speaker: **Jennifer Morse** (Drexel Univ., USA)
Title: An update on the k-Schur approach to statistics problems
Abstract: We will review the k-Schur role in the theory of Macdonald polynomials and talk about some related open problems and new conjectures.

Speaker:**Thomas Lam** (Harvard Univ., USA)
Title:*k-Schur functions and the homology of the affine Grassmannian*
Abstract: I will explain the relationship between Lapointe, Lascoux and Morse's k-Schur functions and the Schubert basis of the homology $H_*(Gr)$ of the affine Grassmannian of $SL(n)$. I will state some general facts about $H_*(Gr)$ then describe Peterson's work on affine Schubert calculus. Peterson's work can be connected to k-Schur functions via the Fomin-Stanley subalgebra and the theory of Stanley symmetric functions.

Speaker: **John Stembridge** (Univ. of Michigan, USA)
Title: *Kostka-Foulkes polynomials of general type and their variations*
Abstract: In this talk we plan to discuss the general features of Kostka-Foulkes polynomials for finite root systems. We will pose several problems or conjectures aimed at developing a general framework for explaining the nonnegativity of their coefficients in a combinatorial way.

If there is time, we will also discuss some additional families of univariate polynomials that also occur in representation theoretic contexts and have the same combinatorial flavor– one related to the Blattner multiplicity formula, and another related to Demazure modules.

Speaker: **Iain Gordon** (University of Edinburgh, United Kingdom)
Title: *Rational Cherednik algebras, diagonal coinvariants, and other animals*
Abstract: I will explain how the representation theory of rational Cherednik algebras is used to get a handle on diagonal coinvariants for Weyl groups. This is quite well understood, but may only be part of a broad scheme. Beyond diagonal invariants there is a dream that the representation theory could shed new light on the n! theorem and its conjectural generalisations to wreath products.

Speaker: **Bogdon Ion** (Univ. of Pittsburgh, USA)
Title: *Nonsymmetric Macdonald polynomials and applications*
Abstract: I will give a quick survey of nonsymmetric Macdonald polynomials and their properties and I will also describe some of their applications (geometric formulas for weight multiplicities and random walks on buildings).

Speaker: **Sarah Mason** (Davidson College, USA)
Title: *A specialization of nonsymmetric Macdonald polynomials*
Abstract: The nonsymmetric Macdonald polynomials can be specialized to polynomials which decompose the Schur functions. We describe several combinatorial properties of these polynomials and their connections to Demazure characters. We discuss a related family of polynomials called "key polynomials" and two new methods for constructing key polynomials.

Speaker: **Adriano Garsia** (Univ. California at San Diego, USA)
Title: *Constant terms and Kostka-Foulkes Polynomials*
Abstract: A problem that arose in Gauge Theory led us to the evaluation of a constant term with a variety of ramifications into several areas from Invariant Theory, Representation Theory, the Theory of Symmetric-Functions and Combinatorics. A significant by-product of our evaluation is the construction of a trigraded Cohen Macaulay basis for the Invariants under an action of SLn(C) on a space of 2n + n2 variables.

Speaker: **Mike Zabrocki** (York University, Canada)
Title: *Combinatorial aspects of generalized Hall-Littlewood symmetric functions*
Abstract: We overview a number of open problems involving the combinatorics of generalized Hall-Littlewood polynomials.

Speaker: **Nantel Bergeron** (York University, Canada)
Title: $\nabla\, k\, \Lambda$
Abstract: We present a series of problems related to ∇ applied to symmetric functions. We show that the analogue results are true for non-commutative symmetric functions.

Speaker: **Tom Koornwinder** (University of Amsterdam, Netherlands)
Title: *The relationship between Zhedanov's algebra AW(3) and DAHA for Askey-Wilson*
Abstract: Zhedanov's algebra $AW(3)$ will be considered with explicit structure constants such that, in the basic representation, the first generator becomes the second order q-difference operator for the Askey-Wilson polynomials. This representation is faithful for a certain quotient of $AW(3)$ such that the Casimir operator is equal to a special constant. A central extension of this quotient of $AW(3)$ can be embedded in the double affine Hecke algebra (DAHA) by means of the faithful basic representations of both algebras. Next I will discuss the relationship between $AW(3)$ and the spherical subalgebra of the DAHA for Askey-Wilson. This one-variable exercise should be a stepping stone for exploring analogues of $AW(3)$ in higher rank.

Abstracts for Posters

Presenter(s): **J. Haglund** (Univ. of Pennsylvania) and **L. Stevens** (UC San Diego)
Title: An extension of the Foata map to standard Young tableaux
Abstract: We define an inversion statistic on standard Young tableaux. We prove that this statistic has the same distribution over SY T () as the major index statistic by exhibiting a bijection on SY T () in the spirit of the Foata map on permutations.

Presenter(s): **L. Tevlin** (Yeshiva Univ.)
Title: Noncommutative Hall-Littlewood Polynomials and q-Cauchy Identity
iAbstract: This poster will contain a proposal for a noncommutative version of Hall-Littlewood (H-L) polynomials. These seem to be natural analogs of classical objects as ribbon H-L polynomials interpolate between ribbon Schur functions and noncommutative monomial symmetric functions, while fundamental H-L polynomials interpolate between noncommutative fundamental and monomial symmetric functions.

Presenter(s): **Alex Woo** (UC Davis)
Title: Garnir modules, Springer fibers, and Ellingsrud-Stromme cells on the Hilbert scheme
Abstract: We calculate defining ideals for certain Sn invariant subspace arrangements of the braid arrangement and relate them (in part using duality) to the cohomology rings of Springer fibers as studied by Garsia and Procesi. This allows us to calculate their graded characters to be particular sums of Hall-Littlewood polynomials. We also relate these subspace arrangements to closed unions of cells on the Hilbert scheme. This is joint work with Mark Haiman.

4 Scientific Progress Made

In the spring of 2007 there was a workshop at the Center de Rescherches Mathématiques (CRM) in Montréal on Combinatorial Hopf algebras and Macdonald polynomials. Many of the speakers at this workshop were also here at the BIRS workshop, and there were benefits from being able to meet again a few months later. It turns out there was quite a bit of significant progress made in the study of Macdonald polynomial combinatorics in the intervening time. For example, Sami Assaf announced that she had successfully completed her ambitious program of trying to prove Schur positivity of LLT polynomials by a combinatorial construction, and she gave a nice presentation of her result at BIRS. This gives the first combinatorial proof of Macdonald's positivity conjecture, and further analysis of her algorithm will undoubtedly lead to exciting new identities for the q, t-Kostka coefficients. In addition, since the conjecture in [21] for the character of DH_n can be expressed as apositive sum of LLT polynomials, Assaf's work also gives an interpretation for the Schur coefficients in this character. Another significant development made in the interim was the discovery by Loehr and Warrington of a conjectured expression for the monomial expansion of the ∇ operator applied to any Schur function, which also made an exciting presentation.

The various researchers at the BIRS workshop all had different points of view on Macdonald polynomials, and it was a joy to hear about all the varioius avenues of research where important applications of Macdonald polynomials arise. The general attitude of the participants seemd to be quite positive about the experience and everyone was happy they attended the workshop. A number of collaborations were begun or enhanced during this time, for example J. Haglund, S. Mason, S. van Willigenburg continued to build on a collaboration started during the CRM workshop. This eventually led to a conjectured generalization of the Littlewood-Richardson rule connected to the study of a new basis for the riung of quasisymmetric functions, a conjecture which was proved by Haglund, Mason, van Willigenburg, and K. Luoto during a recent week long stay at BIRS as part of the Focused Research Group program.

References

[1] F. Bergeron, *Algebraic Combinatorics and coinvariant spaces*, CMS Treatises in Mathematics, Canadian Mathematical Society, Ottawa, ON (2009), 221 pages.

[2] F. Bergeron, A. Garsia, M. Haiman, and G. Tesler, *Identities and positivity conjectures for some remarkable operators in the theory of symmetric functions*, Methods Appl. Anal., **6** (1999), 363–420.

[3] N. Bergeron, F. Descouens, and M. Zabrocki, *A filtration of (q, t)-Catalan numbers*, Adv. in Appl. Math., **44** (2010), 16–36.

[4] I. Cherednik, *Double affine Hecke algebras and Macdonald's conjectures*, Ann. of Math., **141** (1995) 191–216.

[5] I. Cherednik, *Macdonald's evaluation conjecture and difference Fourier transform*, Invent. Math., **122** (1995), 119–145.

[6] I. Cherednik, *Nonsymmetric Macdonald polynomials*, Internat. Math. Res. Notices, (1995), 483–515.

[7] I. Cherednik, *Diagonal coinvariants and double affine Hecke algebras*, Internat. Math. Res. Notices, (2004), 769–791.

[8] A. Garsia and J. Haglund, *A positivity result in the theory of Macdonald polynomials*, Proc. Nat. Acad. Sci. U.S.A., **98** (2001), 4313–4316.

[9] A. Garsia and J. Haglund, *A proof of the q,t-Catalan positivity conjecture*, Discrete Math., **256** (2002), 677–717.

[10] A. Garsia and M. Haiman, *A graded representation model for Macdonald's polynomials*, Proc. Nat. Acad. Sci. U.S.A, **90** (1993), 3607–3610.

[11] A. Garsia and M. Haiman, *A remarkable q,t-Catalan sequence and q-Lagrange inversion*, J. Algebraic Combin., **5** (1996), 191–244.

[12] A. Garsia, M. Haiman, and G. Tesler, *Explicit plethystic formulas for Macdonald q,t-Kostka coefficients*, Sém. Lothar. Combin., **42** (1999), Art. B42m, 45 pp. (electronic).

[13] I. Gordon, *On the quotient ring by diagonal coinvariants*, Invent. Math.,**153** (2003), 503–518.

[14] J. Haglund, *Conjectured statistics for the q,t-Catalan numbers*, Adv. Math., **175** (2003), 319–334.

[15] J. Haglund, *The q,t-Catalan Numbers and the Space of Diagonal Harmonics*, AMS University Lecture Series, 2008.

[16] M. Haiman, *Conjectures on the quotient ring by diagonal invariants*, J. Algebraic Combin., **3** (1994), 695–711.

[17] M. Haiman, *Hilbert schemes, polygraphs, and the Macdonald positivity conjecture*, J. Amer. Math. Soc., **14** (2001), 941–1006.

[18] *Vanishing theorems and character formulas for the Hilbert scheme of points in the plane*, Invent. Math., **149** (2002), 371–407.

[19] J. Haglund, M. Haiman, and N. Loehr, *A combinatorial formula for Macdonald polynomials*, Jour. Amer. Math. Soc. **18** (2005), 735-761.

[20] J. Haglund, M. Haiman and N. Loehr, *A combinatorial formula for Non-symmetric Macdonald polynomials*, Amer. J. Math., **103** (2008), 359-383.

[21] J. Haglund, M. Haiman, N. Loehr, J. B. Remmel, and A. Ulyanov *A combinatorial formula for the character of the diagonal coinvariants*, Duke Math. J., **126** (2005), 195–232.

[22] J. Haglund, K. Luoto, S. Mason, and S. van WIlligenburg, *Quasisymmetric Schur functions*, J. Combin. Theory Ser. A, to appear. See arXiv:0810.2489.

[23] J. Haglund, K. Luoto, S. Mason, and S. van WIlligenburg, *Refinements of the Littlewood-Richardson rule*, Trans. Amer. Math. Soc., to appear. See arXiv:0908.3540.

[24] F. Knop and S. Sahi, *A recursion and a combinatorial formula for Jack polynomials*, Invent. Math., **128** (1997), 9-22.

[25] L. Lapointe, A. Lascoux, and J. Morse, *Tableau atoms and a new Macdonald positivity conjecture*, Duke Math. J., **116** (2003), 103–146.

[26] L. Lapointe, and J. Morse, *Schur function analogs for a filtration of the symmetric function space*, J. Combin. Theory Ser. A, **101** (2003), 191–224.

[27] L. Lapointe, and J. Morse, *Schur function identities, their t-analogs, and k-Schur irreducibility*, Adv. Math., **180** (2003), 222-247.

[28] L. Lapointe, and J. Morse, *A k-tableau characterization of k-Schur functions*, Adv. Math., **213** (2007), 183-204.

[29] L. Lapointe, and J. Morse, *Quantum cohomology and the k-Schur basis*, Trans. Amer. Math. Soc., **360** (2008), 2021–2040.

[30] A. Lascoux and M-P. Schützenberger, *Keys* & *standard bases*, Invariant Theory and Tableaux (Minneapolis, MN, 1988), 125–144, IMA Vol. Math. Appll., **19**, Springer, NY, 1990.

[31] C. Lenart, *On combinatorial formulas for Macdonald polynomials*, Adv. Math., **220** (2009), 324–340.

[32] N. Loehr, and G. Warrington, *Int. Math. Res. Not. IMRN*, Art. ID rnm 157 (2008), 29 pp.

[33] I. G. Macdonald, *A new class of symmetric polynomials*, Actes du 20^e Séminaire Lotharingien, Publ. Inst. Rech. Math. Av., **372** (1988).

[34] I. G. Macdonald, *Symmetric Functions and Hall Polynomials*, Oxford Mathematical Monographs, second ed., Oxford Science Publications, The Clarendon Press Oxford University Press, New York, 1995.

[35] I. G. Macdonald, *Affine Hecke algebras and orthogonal polynomials*, Séminaire Bourbaki, Vol. 1994/95, Astérisque (1996), Exp. No. 797, 4, 189–207.

[36] D. Marshall, *Symmetric and nonsymmetric Macdonald polynomials*, On combinatorics and statistical mechanics, Ann. Comb., **3** (1999), 385–415.

[37] S. Mason, *A decomposition of Schur functions and an analogue of the Robinson-Schensted-Knuth algorithm*, Sém. Lothar. Combin. **57** (2006/08), Art. B57e, 24 pp.

[38] S. Mason, *An explicit construction of type A Demazure atoms*, J. Algebraic Combin., **29** (2009), 295-313.

[39] A. Ram, and M. Yip, *A combinatorial formula for Macdonald polynomials*, arXiv:0803.1146.

[40] V. Reiner and M. Shimozono, *Key polynomials and a flagged Littlewood-Richardson rule*, J. Combin. Theory Ser. A, **70** (1995), 107-143.

5 Participants

Allen, Ed (Wake Forest University)
Assaf, Sami (MIT)
Bandlow, Jason (University of California, San Diego)
Bergeron, Francois (Universit du Quebec a Montral)
Bergeron, Nantel (York University)
Biagioli, Riccardo (Universite Claude Bernard Lyon I)
Can, Mahir (University of Western Ontario)
Descouens, Francois (Universite de Marne-la-Vallee)
Fishel, Susanna (Arizona State University)
Garsia, Adriano (University of California, San Diego)
Gordon, Iain (University of Edinburgh)
Haglund, Jim (University of Pennsylvania)
Hivert, Florent (University of Rouen)
Ion, Bogdan (University of Pittsburgh)
Jing, Naihuan (North Carolina State University)
Kasatani, Masahiro (Kyoto University
Koornwinder, Tom (KdV Institute for Mathematics, University of Amsterdam)
Lam, Thomas (University of Michigan)

Lapointe, Luc (Universidad de Talca)
Li, Huilan (Drexel University)
Loehr, Nick (Virginia Tech)
Mason, Sarah (Wake Forest University)
Morse, Jennifer (Drexel University)
Remmel, Jeff (University of California, San Diego)
Schilling, Anne (University of California, Davis)
Schlosser, Michael (University of Vienna)
Shimozono, Mark (Virginia Tech)
Stembridge, John (University of Michigan)
Stevens, Laura (University of California, San Diego)
Stump, Christian (LACIM)
Suzuki, Takeshi (Okayama University)
Tevlin, Lenny (Yeshiva University)
Thiery, Nicolas M. (Univ Paris-Sud)
van Willigenburg, Stephanie (University of British Columbia)
Vazirani, Monica (University of California, Davis)
Woo, Alexander (St. Olaf)
Yoo, Meesue (University of California, San Diego)
Zabrocki, Mike (York University)