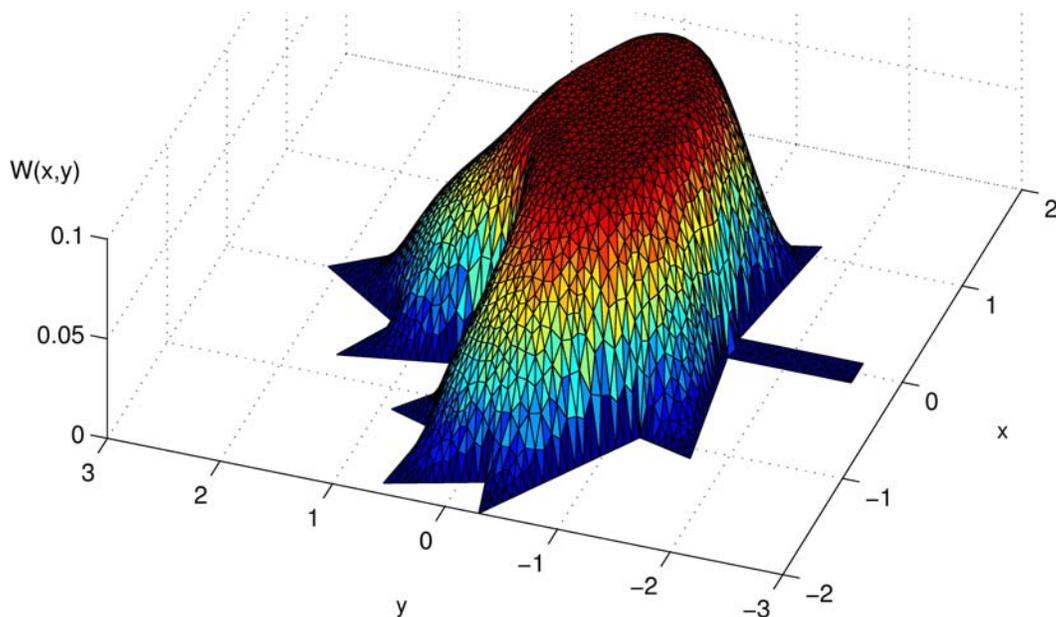




Visco-plastic Fluids, from Theory to Application

October 23rd - 27th, 2005

Banff International Research Station,
Banff, Alberta, Canada





Visco-plastic fluids, from theory to application

October 23rd – 27th, 2005



Welcome to: “Visco-plastic fluids, from theory to application” and thank you for coming to Banff to participate and share your experience.

We hope over the next 5 days to cover a wide range of mathematical and physical problems related to yield stress fluids, to gain a broad perspective of the state of the art in modeling, analysis, computation and experimentation with yield stress fluids, solution of applied problems and various related topics. We hope you enjoy the presentations and that you take the opportunity to make new friends and research collaborators.

This workshop would not be possible without the financial support from the Banff International Research Station, provision of their facilities and the professional support from their staff. We are very grateful for this support.

Neil Balmforth
Ian Frigaard

MEALS

Breakfast (Continental): 7:00 – 9:00 am, 2nd floor lounge, Corbett Hall, Sunday – Thursday

***Lunch (Buffet): 11:30 am – 1:30 pm, Donald Cameron Hall, Sunday – Thursday**

***Dinner (Buffet): 5:30 – 7:30 pm, Donald Cameron Hall, Saturday – Wednesday**

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

*Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am – 12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

Saturday

**16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)
Lecture rooms available after 18:00 (if desired)**

17:30-19:30 Buffet Dinner, Donald Cameron Hall

**20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks available on a cash honour-system basis.**

Meeting schedule: Sunday October 23, 2005

09:15-09:30	Welcome	N. Balmforth, I. Frigaard, BIRS Station Manager
09:30-10:30	Plenary lecture:	<i>“Yield stress phenomena and known solutions of viscoplastic flows.”</i> J. Tsamopoulos, University of Patras
10:30-11:00	Refreshments	
11:00-12:30	Invited contributions	<i>“Auto-plugging properties of drilling mud in fractured wells. Invasion of yield stress fluids in radial flow between two parallel plane”</i> Y. Peysson, Institut Français du Pétrole
		<i>“Flow of Herchel-Bulkley materials in a simple cavity: effects of the yield stress”</i> A.N. Alexandrou/G.C. Georgiou, University of Cyprus
		<i>“Remarks on viscometric and non-viscometric flows of viscoplastic liquids”</i> P. de Souza Mendes, PUC, Brazil
		<i>“Consistency relations in the steady non-isothermal shearing flows of a fluid with a viscosity depending on shear rate, pressure and temperature”</i> R.R. Huilgol, Flinders University of South Australia
		<i>“Application of the DLM/FD and augmented Lagrangian methods for the simulation of steady flow of a Bingham fluid in an eccentric annular geometry”</i> A. Wachs, Institut Français du Pétrole
12:00-14:00	Lunch	
14:00-15:00	Plenary lecture	<i>“Analytical methods for yield stress materials”</i> R. Craster, Imperial College
15:00-15:30	Refreshments	
15:30-17:00	Invited Contributions:	<i>“Quicksand”</i> D. Bonn, ENS Paris/WZI Amsterdam
		<i>“Finite stopping times in Couette and Poiseuille flows of viscoplastic fluids: theory and simulations”</i> E. Mitsoulis, National Technical University of Athens
		<i>“Squeeze flow of compressible pastes”</i> J. D. Sherwood, Schlumberger Cambridge Research, Cambridge, U.K.
		<i>“Rheological characterization of a solder paste for surface mount applications”</i> M.-C. Heuzey, Ecole Polytechnique, Montreal
		<i>“A Bingham viscoplastic in the stress-velocity setting”</i> V. Shelukhin, Lavrentyev Institute of Hydrodynamics
17:00-18:00	Plenary lecture	<i>“Visco-plastic fluids in porous media: physics, mathematics and applications”</i> V. Entov, Russian Academy of Sciences Moscow

Meeting schedule: Monday October 24, 2005

- 09:00-10:00 Plenary lecture: *“Rigid visco-plastic fluids: a solid mechanics perspective”*
I. R. Ionescu, CNRS & University of Savoy
- 10:00-10:30 Refreshments
- 10:30-12:00 Invited contributions
“Difficulties of cutting removal in inclined and horizontal wells for oil field drilling”
Y. Peysson, Institut Français du Pétrole
- “Parameter estimation of thixotropic Herschel-Bulkley fluids”*
A.N. Alexandrou/G.C. Georgiou, University of Cyprus
- “Visco-plastic lubrication, a nonlinearly stable multilayer shear flow”*
I.A. Frigaard, University of British Columbia
- “Plug breaking and wall-layer formation in a wavy walled channel flow”*
A. Putz, University of British Columbia
- “A posteriori error estimates for stationary slow flows of power-law fluids”*
M. Bildhauer, Saarland University
- 12:00-14:00 Lunch
- 14:00-15:00 Plenary lecture *“Yield stress and thixotropy: on the difficulty of measuring a yield stress in practice”*
D. Bonn, ENS Paris/WZI Amsterdam
- 15:00-15:30 Refreshments
- 15:30-17:00 Invited Contributions
“Laminar-turbulent transition of yield stress fluid flow in a pipe” C. Nouar, CNRS-LEMETA, Nancy
- “Particle tracking microrheology of Carbopol”*
J.R. de Bruyn, University of Western Ontario
- “Microrheology of Carbopol dispersions from dynamic light scattering measurements”*
B.J. Frisken, Simon Fraser University
- “Gravity-driven draining of a rivulet of a viscoplastic material down a slowly varying substrate”*
S.K. Wilson, University of Strathclyde
- “Numerical simulation of weakly compressible viscoplastic waxy crude oil flows”*
A. Wachs, Institut Français du Pétrole
- 17:00-18:00 Plenary lecture *“Towards a constitutive law for dense granular flows”*
O. Pouliquen, CNRS-University of Provence

Meeting schedule: Tuesday October 25, 2005

- 09:00-10:00 Plenary lecture: *"THE YIELD STRESS MYTH?" - 20 years on"*
H. Barnes, University of Wales
- 10:00-10:30 Refreshments
- 10:30-12:00 Invited contributions
- "From Flow to Tear in a Micellar Gel"*
A. Belmonte, Pennsylvania State University
- "How to arrest and deflect snow avalanches"*
A. J. Hogg, University of Bristol
- "Reaction-diffusion models for wormlike micellar fluids"* A. Belmonte, Pennsylvania State University
- "Propagation of the interface in a fluid suspension after the onset of shear flow"*
C. Nouar, CNRS-LEMETA Nancy
- "Flow of viscoplastic liquids through axisymmetric expansions-contractions"*
M. Naccache, Pontifícia Universidade Católica
- 12.00-14.00 Lunch
Afternoon
- Open

Meeting schedule: Wednesday October 26, 2005

- 09:00-10:00 Plenary lecture: *"Viscoplasticity and geophysical flows: from field evidence to laboratory experiments"*
C. Ancey, Ecole Polytechnique Federale de Lausanne
- 10:00-10:30 Refreshments
- 10:30-12:00 Invited contributions
- "Long time extrusions of shallow viscoplastic fluids"*
R. Sassi, Università di Milano
- "Multiple solutions of steady granular chute flows"*
A. J. Hogg, University of Bristol
- "Conditions for static bubbles in viscoplastic fluids"*
N. Dubash, Imperial College
- "Rheology and fluid mechanics of two-phases food fluids within circular pipes. Part A: General properties and rheological flow curves"*
P. Perona, Swiss Federal Institute of Technology
- "Rheology and fluid mechanics of two-phases food fluids within circular pipes. Part B: Laminar-turbulent transition and related resistance laws"*
P. Perona, Swiss Federal Institute of Technology
- 12.00-14.00 Lunch

14.00-15.00	Plenary lecture	<i>“On the numerical simulation of visco-plastic flow, a la Bingham”</i> R. Glowinski, University of Houston
15.00-15.30	Refreshments	
15.30-17.00	Invited Contributions	<i>“Rayleigh-Benard stability of a Bingham fluid - some initial results”</i> I.A. Frigaard/D. Vola, University of British Columbia/IRSN, France <i>“Rayleigh-Benard-Poiseuille instability of Bingham fluid”</i> C. Metivier/C. Nouar, LEMTA, Nancy <i>“Viscoplastic, surface-tension-driven fingering”</i> S. Ghadge/N. Balmforth, University of British Columbia <i>“Roll waves in Mud”</i> N. Balmforth, University of British Columbia

Thursday October 27, 2005: Discussion sessions, to be announced

Abstracts

“Yield stress phenomena and known solutions of visco-plastic flows.”

J. Tsamopoulos and Yannis Dimakopoulos,
University of Patras

The distinguishing property of viscoplastic liquids is their yield stress, the stress level below which these liquids do not flow and above which they flow with a viscosity that is a function of the shear rate. The surface at which this solid to liquid transition occurs is called the yield surface. This seemingly "innocent" departure from the Newtonian behaviour makes computations of flows with viscoplastic fluids in two and three dimensions quite involved, because, at the yield surface, the viscosity becomes singular and special techniques must be introduced. Moreover, the location and shape of the yield surface is *a-priori* unknown, increasing the difficulty of solving such problems.

Two types of methods have been developed to deal with this problem: the augmented Lagrangian method and the regularization method, according to which the singularity in the viscosity function is somehow removed. The most popular regularization methods are those proposed by Bercovier and Engelman (J. Comp. Phys. 36, (1980)), which adds a small parameter to the shear rate arising in the denominator of the viscosity function and by Papanastasiou (J. Rheol. 31, (1987)), which approximates the viscosity by an exponential function approaching infinity (and solid behaviour) only asymptotically. The first one still needs to determine the yield surface together and with the same accuracy as the flow field. On the other hand, following Papanastasiou's idea, the flow field is calculated first and the yield surface subsequently and, could produce less accurate results, if the value of the model parameter in the exponent is not carefully chosen.

We will present steady and transient solutions of problems treated with either one of these two regularization methods in combination with the finite element/Galerkin method for solving the relevant PDEs. When the yield surface is to be determined along with the flow field or when also a free/moving boundary arises in the problem and deforms because of the flow, an algebraic or (preferably) an elliptic mesh generation method must be employed. We have developed such a robust and flexible mesh generation scheme [Dimakopoulos and Tsamopoulos, J. Comp. Phys. 192, (2003)] and employed it in several such problems. The flow field equation set is decoupled from the mesh generating equation set. Solution for the two sets of unknowns is obtained using Picard iterations between them and Newton/Raphson iterations within each set.

“Auto-plugging properties of drilling mud in fractured wells. Invasion of yield stress fluids in radial flow between two parallel plane”

Y. Peysson and B. Herzhaft, Institut Français du Pétrole

Drilling muds are complex non-Newtonian fluids used to drill oil wells. The drilling of naturally fractured reservoirs may lead to significant mud losses that are problematic for drilling operators (Non-Producing Time, high cost of the drilling fluids). Mud losses in natural fractures are characterized by a typical mud loss history curve determined by following the mud pit level evolution. This curve starts with a high mud loss rate (quick decrease of the mud pit level) as the drilling bit hits the fracture and the mud starts to invade the fracture, then the mud loss rate decreases and finally stops due to the fluid yield stress. This "auto-plugging" property can be very convenient to limit mud losses in difficult areas.

Drilling through naturally fractured areas may lead to significant losses of fluids into the fractures can be observed, and this fluid invasion need to be controlled. We address experimentally this problem of fracture invasion by drilling fluids assuming for the natural fracture a simplified geometry of a slot of constant width intercepting a circular well. We experimentally study the evolution of the radius of fluid invasion with time for different configurations and different fluid rheological properties. An invasion model is proposed to give a relation between the fluid rheology and the invasion of the fracture. An experimental set up was built consisting of two parallel square plates connected to a valve and a reservoir. Hydrostatic pressure is controlled by the height of liquid in the reservoir. A camera and an image processing system is used to monitor the radial flow when the valve is open. The radius of invasion with time is then calculated from the picture analysis software. Different fluid systems were tested, from Newtonian glycerol solutions, to a bentonite dispersion in water (this latter fluid being commonly largely used in the drilling industry). The properties of this last fluid is time dependent and present a yield stress in certain conditions. We showed that for the bentonite solution, a maximum radius is reached and the flow stopped. The system presents an "auto plugging" properties property which is consistent with the classical yield stress measurements results. We analyzed the relation between radius and flow properties and the time dependent behavior of the fluid is discussed.

“Flow of Herchel-Bulkley materials in a simple cavity: effects of the yield stress”

Andreas N. Alexandrou, Philippe Le Menn & Georgios C. Georgiou
University of Cyprus

The purpose of this work is to address the overall issue of the reliability of the semisolid metal process (SSMP) and its ability to produce consistently high quality parts. Due to the unique rheology of semisolid suspensions and depending on the slurry properties, dies fill in very distinct ways. Eventually, the various filling patterns have direct implications on the quality and mechanical properties of the produced parts.

In this paper we show experimental results that are typical of those that lead to questions about the consistency of the process. While the rheology of SSM suspensions is not fully understood, we demonstrate through proper modelling and simulations that the results can be explained by considering the finite yield stress of the slurries. This implies that by controlling the process parameters the instabilities can be avoided. We also provide stability maps to indicate the parameter space where unstable behavior is more likely to occur in a simple rectangular 2-D cavity.

“Remarks on viscometric and non-viscometric flows of viscoplastic liquids”

P.R. de Souza Mendes,
Pontifícia Universidade Católica, Rio de Janeiro, Brazil

Firstly we employ a recently proposed viscosity function (Souza Mendes and Dutra, 2004) to analyze the Couette flow of yield-stress liquids, focusing our attention on the range of shear stress such that here exist both a yielded flow region and an unyielded one. The solution of the momentum equation shows that the shear rate may differ by several orders of magnitude within the flow domain, which is neglected in standard rheometry practices without justification. However, it is shown that this fact is actually of no consequence in the measurement of viscosity, i.e. no shear rate corrections are needed.

For a complex kinematics where the flow is not steady in the Lagrangian sense and both yielded and unyielded flow regions are present, experimental results suggest that the material elasticity that is present before yielding may play an important role and hence should be taken into account, especially in the determination of the position of the yield surfaces. Therefore, for complex flows the generalized Newtonian liquid model is not appropriate for flows of viscoplastic liquids that are not steady in the Lagrangian sense. A simple constitutive equation for viscoplastic liquids that is capable of predicting a viscoelastic behavior below the yield stress is proposed.

Souza Mendes, P. R., Dutra, E. S. S., 2004. Viscosity function for yield-stress liquids. *Applied Rheology* 14 (6), 296-302.

“Consistency relations in the steady non-isothermal shearing flows of a fluid with a viscosity depending on shear rate, pressure and temperature”

R.R. Huilgol and Z. You,
Flinders University of South Australia

Four steady non-isothermal shearing flows of a compressible fluid with a viscosity depending on the shear rate, pressure and temperature are considered. Consistency relations between the pressure gradient in the flow direction and that in the direction of shearing are derived. In particular, the dependence of the viscosity on pressure plays the most significant role in these relations. Examples of pressure fields obeying these compatibility conditions are given and their significance is explored. Extensions of the results to incompressible and viscoplastic fluids are explored. Clearly, failure to satisfy these consistency relations must result in secondary flows.

“Application of the DLM/FD and augmented Lagrangian methods for the simulation of steady flow of a Bingham fluid in an eccentric annular geometry”

A. Wachs and Yannick Peysson,
Institut Français du Pétrole

This paper deals with the numerical simulation of uniaxial steady flows of viscoplastic fluid in an eccentric annular geometry. The situation considered refers to mud flow in drilling operations or cement slurry flow in cementation process in the oil and gas industry. The rheological model retained is the classical isothermal Bingham model. The steady solution of the governing equations is obtained through a transient algorithm that allows to compute solutions at an imposed flow rate. Mathematical difficulties related to the Bingham equation are overcome thanks to the use of Lagrange multipliers and an augmented Lagrangian/Uzawa method. The inner cylinder is treated as a fictitious domain (FD) on which a zero velocity constraint is imposed. The velocity constraint is relaxed by the introduction of distributed Lagrange multipliers (DLM). The governing equations are discretized using a Finite Element Method with triangular elements. The resulting numerical method highlights strong and robust convergence properties. The use of the DLM/FD enables to study the influence of the eccentricity very conveniently. In particular, we show the effect of the mesh refinement on the solution accuracy in relation with the use of the DLM/FD method. Results obtained underline the effects of the eccentricity on the flow physiomy.

“Analytical methods for yield stress materials”

R. Craster,
Imperial College

It is often the case that numerical methods seem the only viable route to solving the highly nonlinear equations that arise during the flow of yield stress (say Bingham/Herschel Bulkley) materials. One has an internal "free" surface, the yield surface, that separates rigid from fluid media and this must be determined: in many ways this feature of the flow makes the subject interesting both physically and mathematically. There are a couple of complementary analytical methods that are worth knowing about and this talk aims to cover them. The first is limited to longitudinal shear flows and is based upon Legendre transformations and the hodograph technique. This involves a transformation of the nonlinear equations to linear ones (this is not a linearisation of the equations, but a true mapping) which can then be solved explicitly and one must then map back to the physical domain. There are sadly further limitations of the method and this too will be discussed, as will interesting and useful connections to other areas (nonlinear filtration and elasticity). Secondly, there is another approach, suitable for free surface flows which are yield stress dominated. In this case the evolving material equations can be reduced to a single nonlinear first order PDE and this then is explicitly solvable using the method of characteristics. These exact solutions then provide useful alternatives to numerics and/or allow one to estimate or check certain features (say yield surfaces near interior corners).

“Quicksand”
D. Bonn,
ENS Paris/WZI Amsterdam

Quicksand is the generic name for unstable soils reputed to trap anyone who treads on it. Popular wisdom has it that one should not move when trapped in quicksand, as motion makes one sink in even deeper and that once trapped, it is difficult to escape. I will provide an explanation for these observations by studying the most commonly encountered form of natural quicksand. We show that a spectacular liquefaction of the material occurs when a stress is applied to the material: the liquefaction is the reason why one sinks away, and it is more pronounced for larger stresses. By constructing "laboratory quicksand", we demonstrate that the liquefaction is due to the structure: quicksand is a loose granular packing of sand particles stabilized by a clay matrix that forms a particulate gel. The stress liquefies the clay matrix, and the granular assembly collapses, expelling water. This results in a densely packed system that is practically impossible to dilate: it is for this reason that once trapped it is difficult to get out of quicksand. A sinking test demonstrates that, due to buoyancy, it is impossible to drown in the quicksand.

“Finite stopping times in Couette and Poiseuille flows of viscoplastic fluids: theory and simulations”

E. Mitsoulis, Ioannis Argyropaidas, Georgios Georgiou,
Maria Chatzimina, and Raj Huilgol

Steady Couette and Poiseuille flows of Bingham materials come to rest in a finite amount of time, if either the applied pressure falls below a critical value, or the moving boundaries are brought to rest. An explicit formula for a bound on the finite stopping time in each case has been derived. This bound depends on the density, the viscosity, the yield stress, a new geometric constant, and the least eigenvalue of the second-order linear differential operator for the interval under consideration.

To solve numerically such problems, several regularized models that overcome the discontinuity of the yield-stress models have been proposed and analyzed over the years. The exponential model is shown to give as good predictions as the ideal Bingham model for many cases of viscoplastic flow.

Numerical simulations with the Bingham-Papanastasiou model have been pursued for Couette and Poiseuille flows (planar, tubular and annular) with the purpose to see whether it is possible to predict the finite stopping times, despite the fact that the model has a nonlinear smooth viscosity function. Our calculations confirm that the stopping times are indeed finite when the yield stress is nonzero. The decay of the volumetric flow rate, which is exponential in the Newtonian case, is accelerated and eventually becomes linear as the yield stress is increased. The results are given as a function of the Bingham number and the Halting number appropriate for cessation of flow.

“Squeeze flow of compressible pastes”

J. D. Sherwood,

Schlumberger Cambridge Research, Cambridge, U.K.

During drilling of an oil well, the permeable rock walls of the wellbore become coated with a paste-like filter cake containing a high volume fraction of solid particles from the drilling fluid [1]. The cake to a first approximation may be considered as a Bingham fluid [2]. When wireline tools are brought into contact with the wellbore wall, the cake is squeezed out of the gap between tool and rock. The compressibilities of both liquid and solid within the cake are small, but the matrix of solid particles can be compacted when liquid is squeezed out of the paste.

I first discuss squeeze flow of a homogeneous compressible material [3], and then consider the effect of motion of liquid relative to solid during squeezing of a paste [4,5].

[1] J.D. Sherwood & G.H. Meeten, *J. Petroleum Science and Engineering* 18 (1997) 73.

[2] J.D. Sherwood, G.H. Meeten, C.A. Farrow & N.J. Alderman, *J. Non-Newtonian Fluid Mech.* 39 (1991) 311.

[3] J.D. Sherwood, *J. Non-Newtonian Fluid Mech.* 112 (2003) 115.

[4] J.D. Sherwood, *J. Non-Newtonian Fluid Mech.* 104 (2002) 1.

[5] J.D. Sherwood, *J. Non-Newtonian Fluid Mech.* 128 (2005) 163.

“Rheological characterization of a solder paste for surface mount applications”

M.-C. Heuzey,

Ecole Polytechnique, Montreal

Solder pastes used in surface mount soldering techniques (SMT) are very complex suspensions containing high volumes of metallic powder in a carrier fluid. The rheological complexity results largely from the carrier fluid itself, which is a suspension of colloidal particles. In this work, we have characterized the rheological properties of a typical carrier fluid and its solder paste containing 64 vol. % metallic powder. A six-blade vane geometry was used to avoid wall slip and sample fracture. All measurements were carried out following pre-shearing and rest time in order to obtain reproducible results. Steady shear experiments showed that the solder paste was highly shear thinning and thixotropic. In oscillatory shear, the linear viscoelastic domain was found to be very narrow for both the suspending fluid and the paste. Frequency sweep tests in the linear domain revealed a gel-like structure with a nearly constant G' for the suspending fluid and a slightly increasing G' for the solder paste. From creep experiments, a yield stress of about 40 Pa was determined for the suspending fluid at temperatures between 25C-40C, and of 100 Pa at 4C. A much larger yield stress, 480 Pa, was determined for the solder paste at 25C.

“A Bingham viscoplastic in the stress-velocity setting”

V. Shelukhin,

Lavrentyev Institute of Hydrodynamics

A new stress-velocity formulation is proposed for the equations of the Bingham viscoplastic. The approach is alternative to the Duvaut-Lions variational inequality. Global existence and uniqueness theorems are proved by a smooth approximation of the discontinuous Bingham stress-strain law. In fact, we treat the Bingham viscoplastic as a limit of nonlinear fluids. The equations proposed allow for control of the rigid-fluid interface. A free-boundary problem is formulated to describe the evolution of the interphase surface. A dimensionless parameter is introduced to decide when a rigid piece of Bingham viscoplastic begins to flow.

[1.] V. V. Shelukhin. Bingham viscoplastic as a limit of non-Newtonian fluids, *Journal of Mathematical Fluid Mechanics*, 4 (2002), 109-127.

[2.] I. V. Basov and V.V. Shelukhin. Generalized solutions to the equations of Bingham compressible flows, *Z. Angew. Math. Mech.*, 78 (1998), 1-8.

“Visco-plastic fluids in porous media: physics, mathematics and applications”

V. Entov,

Russian Academy of Sciences Moscow

Flow of visco-plastic fluids through porous media occurs in many processes of petroleum engineering. The most significant effects are related to development of petroleum reservoirs containing crude oil having non-zero yield stress in situ. While occurrence of such crudes is still a subject of discussion, there are multifarious evidences that such oils do exist, and presence of yield stress may affect significantly all processes of oil recovery. Their prediction and description leads usually to free-boundary problem, many of which can be effectively solved analytically. The talk discusses some approaches, results and applications of related theory, as well as open problems and conclusions with respect to general flows of Bingham fluids.

“Rigid visco-plastic fluids: a solid mechanics perspective”

I.R. Ionescu,

CNRS & University of Savoy

In the first part a general methodology for constructing rigid visco-plastic fluid-type constitutive equations is presented. Starting from classic yield conditions for plastic solids, and neglecting the elastic effects, compressible or incompressible rigid visco-plastic fluid models are obtained using two methods: the visco-plastic regularization and the stress superposition method. Examples of constitutive equations obtained using both procedures for the description of the behavior of porous media when subjected to large deformations and high strain rates are presented. I shall point out that irrespective of the method used, it is not possible to obtain consistent visco-plastic fluid formulations starting from certain yield functions.

In the second part I shall analyze the onset of the flow of a rigid visco-plastic fluid through a limit load analysis. To be simple only the 2-D unidirectional flow of a Bingham fluid will be considered, but I shall give some numerical results concerning the plane flow of a more general fluid. The blocking property, characterized by the safety factor, is connected to two optimization problems in terms of velocities and stresses. For the velocity analysis we prove that the minimization problem in $BV(\Omega)$ is equivalent to a shape optimization problem. The optimal set appears to be the part of the fluid which flows whenever the loading parameter becomes greater than the safety factor. The stress approach involves a supremal optimization problem in $W^{1,\infty}(\Omega)$. The $L^p(\Omega)$ approximation technique is used to get a sequence of minimum problems for smooth functionals. Two numerical approaches, following these two analysis, are used to compute the safety factor for some applications in landslides modeling.

In the third part I shall consider the flows involving high rate deformations zones coupled with rigid ones. A mixed finite-element and finite-volume strategy is developed for numerical modeling of the flow of a compressible rigid viscoplastic fluid. This numerical method is used to model the high speed penetration of a rigid projectile into geological and cementitious targets. It accurately describes the density changes around the projectile, the stress field as well as the shape and location of the deformation zone (viscoplastic region) in the target.

“Difficulties of cutting removal in inclined and horizontal wells for oil field drilling”

Y. Peysson,
Institut Français du Pétrole

The new well configurations involve long horizontal and highly deviated sections. Hole cleaning and cuttings transport are among the main difficulties related to these drilling techniques. Poor hole cleaning leads to high torque and drag, risk of blockage and even loss of the well. All these phenomena are to be feared when drilling such wells. A better comprehension of the mechanisms controlling the cutting transport in drilling muds is needed.

Drilling muds are complex fluids made with many different components. Two main families can be found depending on the based fluid: water based muds (WBM) and oil based muds (OBM). Water based muds are mainly clays suspensions with polymers additives; oil based muds are emulsions of brine (around 30% in volume) in an oil phase with surfactants, polymers and diverse chemicals additives.

Drilling muds exhibit a non-Newtonian behavior: a shear-thinning viscosity and yield stress. These complex rheological fluids are in general modeled by a Herschel-Bulkley constitutive equation. But strong time effects are also involved in such fluid. Different experimental studies related to hole cleaning will be discussed. Experimental removal of controlled quantities of cuttings in an annular geometry have been performed in inclined tubes showing the strong effect of inclination on the cutting removal efficiency. Studies of the settling behavior of solid spherical particle in polymer and clay solutions will also be discussed.

“Parameter estimation of thixotropic Herschel-Bulkley fluids”

A.N. Alexandrou and G.C. Georgiou,
University of Cyprus

Semisolid slurries are a mixture of rounded, rosette-like solid particles and liquid at a temperature between the liquidus and solidus limits. The average solid volume fraction is a function of the bulk temperature of the suspension and varies from zero to unity depending on the local temperature. During processing the viscosity changes significantly due to the evolution of the internal structure. In general, semisolid materials behave as viscoplastic fluids characterized by a finite yield stress and by material properties that are time- and shear-rate depended. Therefore, semisolid slurries are modeled as Herschel-Bulkley fluids with time-dependent parameters.

An essential element for the integration of the semisolid metal (SSM) process in the production of complex commercial components is the availability of accurate mathematical and computational tools that could describe both the material characteristics and the dynamics of semisolid slurries during die filling. Computational simulation and modeling is also a powerful and an effective tool that can be used to understand complex physical phenomena, such as those observed with semisolid slurries, and to support theoretical models describing them.

We show how important physics can be revealed using experiments combined with numerical simulations. We also investigate in detail the flow in a rotational viscometer which relates to a popular experiment commonly used to determine rheological constants of semisolid slurries. The goal is to use mathematical and computational tools to test postulates for physical and theoretical models used to describe complex semisolid slurry behavior. The ultimate objective of this work is to contribute to the development of a methodology, which relates simulations to actual experimental results in order to determine the material constants of Herschel-Bulkley fluids that exhibit complex thixotropic behavior.

“Visco-plastic lubrication, a nonlinearly stable multilayer shear flow”

Ian Frigaard, Chun Huen, Dominic M. Martinez, Miguel Moyers-Gonzalez
University of British Columbia

A common problem in multi-layer shear flows, especially from the perspective of process engineering, is the occurrence of interfacial instabilities. For purely viscous fluids these occur at both long and short wavelengths, and at low Reynolds numbers. However, multi-layer duct flows can be stabilised by using a suitable yield stress fluid as the lubricant [1,2]. We focus on the simplest practically interesting case of visco-plastically lubricated viscous shear flow: a core-annular pipe flow consisting of a central core of Newtonian fluid surrounded by a Bingham fluid. First we show how interfacial instabilities may be eliminated through the introduction of a yield-stress fluid as the lubricant and by preserving an unyielded layer adjacent to the interface. Second, we show that nonlinear stability of this type of two-layer flow can also be achieved, at significant Reynolds numbers. Finally we show the initial results of an experimental study of these flows. The ability to have stable multi-layer flows at moderate Reynolds numbers opens up new possibilities for high speed manufacture/processing of multi-layered products.

[1] Frigaard, I.A., "Super-stable parallel flows of multiple visco-plastic fluids." J. Non-Newtonian Fluid Mech. 100, pp. 49-76, (2001).

[2] Moyers-Gonzalez, M., Frigaard, I.A. and Nouar, C., "Nonlinear stability of a visco-plastically lubricated shear flow." J. Fluid Mech. 506, pp.117-146, (2004).

“Plug breaking and wall-layer formation in a wavy walled channel flow”

A. Putz & I. Frigaard

University of British Columbia

We show numerical simulation results for the Poiseuille flow of a Bingham fluid down a wavy-walled channel, at long wavelengths. For small amplitude perturbations of the plane channel, the rigid plug region remains intact, but breaks at a critical amplitude. The asymptotic theory in [1] predicts both the shape of the perturbation of the yield surface and the critical amplitude when the plug breaks. Counter-intuitively, the unyielded plug is wider in the narrower part of the channel and narrower in the wider part.

Numerically, this is a challenging problem. For high accuracy in a channel of large aspect ratio, a large number of cells are required. Regularisation methods are inappropriate for such geometries and anyway do not necessarily predict the correct yield surface position. We have used finite elements with the augmented Lagrangian method, coupled with an adaptive meshing technique that focuses the mesh close to the yield surface, see [2].

The numerical results confirm the results of [1]. Unequivocally, this means that the lubrication paradox of [3] is in fact no paradox, i.e. it is simply that appropriate numerical and asymptotic methods must be applied. The numerical results are able to reveal more of the flow structure than the results in [1]. In particular, we determine the shape of the plug region during breaking. We also investigate larger amplitude wall perturbations at significant Bingham numbers, B . Here we show that a unyielded plug begins to form on the wall in the widest part of the channel, at a critical amplitude. It is not apparently necessary for the plug to break, since the static wall-layer blocks the channel allowing a faster plug velocity at the channel centre, which reduces the stresses.

[1] Frigaard I.A. & Ryan D.P., "Flow of a visco-plastic fluid in a channel of slowly varying width", *J. Non-Newtonian Fluid Mech.* **123** (2004) 67-83

[2] N. Roquet, P. Saramito, An adaptive finite element method for viscoplastic fluid flows in pipes, *Comput. Meth. Appl. Mech. Eng.* **190** (2000) 5391-5412.

[3] G.G. Lipscomb, M.M. Denn, "Flow of Bingham fluids in complex geometries", *J. Non-Newtonian Fluid Mech.* **14** (1984) 337-346.

“A posteriori error estimates for stationary slow flows of power-law fluids”

M. Bildhauer, Saarland University

This is a joint work with Martin Fuchs and Sergey Repin.

We consider the stationary Stokes problem for so-called power-law fluids and prove functional type a posteriori error estimates for the difference of the exact solution and any function from the admissible energy class.

“Yield stress and thixotropy: on the difficulty of measuring a yield stress in practice”

D. Bonn,

ENS Paris/WZI Amsterdam

The yield stress of many yield stress fluids has turned out to be difficult to determine experimentally. This has led to various discussions in the literature about those experimental difficulties, and the usefulness and pertinence of the concept of yield stress fluids. I will argue that most of the difficulties disappear when taking the thixotropy of yield stress fluids into account, and will demonstrate an experimental protocol that allows to obtain reproducible results on the critical stress necessary for flow of these fluids. As a bonus, I will show that the interplay of yield stress and thixotropy allows one to account for the ubiquitous localization of shear in these materials. The price to pay, however, is high: due to the thixotropy the yield stress is no longer a material property, since it depends on the (shear) history of the sample.

“Laminar-turbulent transition of yield stress fluid flow in a pipe”

J. Peixinho, C. Nouar, C. Desaubry, B. Theron

The present work is an experimental study of the laminar-turbulent transition flow in a cylindrical pipe for a yield stress fluid. The fluid used is an aqueous solution of 0.2 w% Carbopol. To bring out the effect of the yield stress on the critical conditions, two other fluids were used: Shear thinning fluid without yield stress (aqueous solution of 2% CMC) and a Newtonian fluid (glucose syrup). The rheological properties (simple shear viscosity and first normal stress difference) were obtained using a controlled stress rheometer from TA Instruments. The flow is monitored using pressure drop and (laser Doppler) axial velocity measurements. As expected, it is observed that the yield stress contributes to stabilize the flow. In addition, the critical conditions were compared with different phenomenological criteria. However, here we want to emphasize two particular finds: (i) In the early stage of transition, it has been observed an increase of the fluctuations of the axial velocity outside a central region while the fluctuations remain at a laminar level inside this region. Then, with increasing the Reynolds number, the fluctuations increase in the whole section with the apparition of turbulent spots. (ii) The velocity profiles are symmetric for both laminar and turbulent flow, and present an asymmetry at all stages of transition.

“Particle tracking microrheology of Carbopol”

Felix Oppong and John R. de Bruyn,
University of Western Ontario

We have studied the diffusion of small fluorescent particles suspended in Carbopol, a viscoplastic polymer dispersion. Dynamic light scattering and particle tracking using fluorescence microscopy give consistent results for the averaged motion of an ensemble of suspended particles, and indicate that the diffusion of the particles is restricted by the structure of the fluid. Here we present a more detailed analysis of the particle tracking data. We show that the suspended particles can be divided into different populations which probe different microscopic rheological environments. We determine the values of the viscous and elastic moduli of the material at the microscopic scale, and obtain estimates for the size of the micro-scale inhomogeneities in the material. We compare our microrheological results with bulk rheological measurements.

“Microrheology of Carbopol dispersions from dynamic light scattering measurements”

L. Rubatat, A.E. Bailey and B.J. Frisken
Simon Fraser University

The diffusion of particles in a viscoelastic medium can be studied through the temporal fluctuations of light scattered by the particles. By analyzing correlations in these fluctuations both the mean-squared displacement (MSD) of the particles and, through the generalized Stokes-Einstein relation, the viscoelastic moduli are obtained. We have used this method to study the rheological properties of Carbopol ETD 2050 dispersions at sub-micron length scales. In this paper, we will discuss results obtained by analyzing the light scattered from particles ranging from 50 nm to 1 μ m in diameter and measured at a variety of scattering angles. These results will be compared to those obtained through particle tracking experiments and bulk rheology measurements.

“Gravity-driven draining of a rivulet of a viscoplastic material down a slowly varying substrate”

S.K. Wilson,
University of Strathclyde

We use the lubrication approximation to investigate the steady locally unidirectional gravity-driven draining of a thin rivulet of viscoplastic material, modelled as a biviscosity fluid (or, as a special case, as a Bingham material), down a slowly varying substrate. In contrast to the earlier work on viscoplastic rivulets we consider small-scale flows, such as those found in many industrial coating and printing processes, in which surface-tension effects play a significant role. We interpret our results as describing a slowly varying rivulet draining in the azimuthal direction from the top to the bottom of a large horizontal circular cylinder. Provided that the yield stress is non-zero we find that the flow is always unyielded near the top of the cylinder (where the rivulet becomes infinitely wide in the transverse direction), and, except in the special case when the viscosity ratio is zero, near the bottom of the cylinder (where it becomes infinitely thick). For sufficiently small values of the prescribed volume flux the flow is unyielded everywhere, but for larger values of the flux the flow near the substrate in the centre of the rivulet is yielded. We obtain numerically calculated values of the semi-width of the rivulet and of the yielded region as well as of the maximum height of the rivulet and of the yielded region for a range of parameter values, and describe the asymptotic behaviour of the solution in the limits of large and small yield stress, large and small flux, and small viscosity ratio. In the special case of a Bingham material the flow near the top of the cylinder consists of an infinitely wide rigid and stationary plug, while elsewhere it consists of two rigid and stationary “levees” at the edges of the rivulet and a central region in which the flow near the free surface is a “pseudo-plug” whose velocity does not vary normally to the substrate, separated from the “fully plastic” flow near the substrate by a “pseudo-yield surface”.

- [1] Ross, A.B., Wilson, S.K., Duffy, B.R. “Thin-film flow of a viscoplastic material round a large horizontal stationary or rotating cylinder” *J. Fluid Mech.* 430, 309--333 (2001)
- [2] Wilson, S.K., Duffy, B.R., Ross, A.B. “On the gravity-driven draining of a rivulet of viscoplastic material down a slowly varying substrate” *Phys. Fluids* 14, 555--571 (2002)

“Numerical simulation of weakly compressible viscoplastic waxy crude oil flows”

Guillaume Vinay and Anthony Wachs

Institut Français du Pétrole

In the last decades, the exploration of new oilfields resulted in the production of considerable quantities of non-conventionnal crude oils like waxy crude oils (about 20% of the world petroleum reserves). Handling waxy crude oils under steady flowing conditions is worthwhile because of its low viscosity under fair temperature conditions. For assorted purposes (maintenance, emergency ...) some flow shutdowns may occur and a gel-like structure in the crude oil bulk builds up due to the temperature decrease in the pipeline. In addition to the appearance of non-Newtonian properties (viscoplastic, thixotropic, temperature-dependant) related to this interlocking gel-like structure formed by the paraffin crystals, the waxy crude oil undergoes a thermal shrinkage related to the appearance of gas voids that confers a kind of compressibility to the material. The main difficulty with waxy crude oils transportation is the restartability issue in which the compressibility plays a prevailing role. In this paper we describe isothermal transient flows of a weakly compressible viscoplastic fluid in an axisymmetric pipe geometry. We retain the Bingham model to describe the viscoplastic feature of the fluid and the compressibility is introduced in the continuity equation thanks to the isothermal compressibility coefficient. A particular interest is devoted to the velocity-pressure problem in which the "true" (without regularization procedure) Bingham model is accounted for, thanks to the use of Lagrange multipliers techniques and an augmented Lagrangian/Uzawa method. The mass, momentum and constitutive equations are discretized using a Finite Volume method on a staggered grid with a TVD (Total Variation Diminishing) scheme for the convective terms. The resulting numerical method highlights strong and robust convergence properties. Obtained results regarding the transient solution underline the influence of the compressibility on the flow pattern, especially in terms of yielded/unyielded regions, pressure and time to restart the flow.

“Towards a constitutive law for dense granular flows”

O. Pouliquen,

CNRS-University of Provence

The description of the flow of grains still represents a challenge, as they often occur in a dense regime where the particles interact both through frictional contacts and collisions. However, recent experiments and numerical simulations of dry and dense granular flows suggest that a simple rheological description in terms of a shear rate dependent friction coefficient may be sufficient to capture the major flow properties. We will discuss this idea and will propose an empirical constitutive law which allows quantitative predictions in different flow configurations. The limits of the approach close to the flow threshold will also be discussed.

“THE YIELD STRESS MYTH?” - 20 years on”

H. Barnes,
University of Wales

It has been twenty years since Walters and I published our provocative paper with the challenging but guarded title 'The yield stress myth?' (Barnes H A, Walters K 'The Yield Stress Myth', *Rheologica Acta*, 24 (4): 323-326 1985). Since that time it has been cited at least 160 times (ISI Web of Knowledge) and while initially it caused some serious debate, its rate of citation is now declining. In detail, we note that from the fifth to the fifteenth year after publication the rate averaged around ten per annum, while in the first five years after publication, the number increased to and after fifteen years declined from this average. After twenty years, what is now clear is that the amount of experimental information that supports our original conclusion has increased enormously with the continued use and improvement of controlled stress rheometers able to measure to lower and lower shear stresses and display lower and lower shear (creep) rates. At the same time, the use of the yield stress concept surges on!

However, in a number of other papers published since the original one, I have been careful to state that the use of a yield stress as a mathematical convenience to define part of a flow curve has never been in dispute, but that the simplistic definition of a physical parameter is not tenable, i.e., that there is no flow below the defined yield stress.

Of course it depends on what one means by 'flow'. If we extend the definition to 'observable or visible flow', then I would have no argument at all with such a proposition and we could all agree! As most mathematical uses of viscoplastic theories are actually used to predict such 'observable or visible flow', usually for engineering purposes with short timescales, then it is worthwhile investigating what are the best kinds of theories that incorporate yield stresses: slump tests for concrete or chocolate thickness in coating machines being typical examples. However, experimentalists who measure creep below the yield stress are able to provide data that is useful over very long time scales, such as shelf life of months or years, as for instance when the physical stability of a multiphase liquid or paste is important.

“From Flow to Tear in a Micellar Gel”

A. Belmonte and Josh Gladden
Pennsylvania State University

We present an experimental study of the motion of a cylindrical rigid rod through a layer of gel-like micellar surfactant material. We observe distinct material responses depending on the rod size and velocity, including fluid-like flow and a solid-like tearing. Optical birefringence techniques are used to measure the stress field due to the rod motion, which takes on an asymmetric dipolar birefringent pattern modified by the presence of tearing in the material. Due to the intrinsic viscoelasticity of the gel, the tearing pattern heals in the far field wake of the rod, and after a very long time the layer returns to its original state.

“How to arrest and deflect snow avalanches”

A. J. Hogg,
University of Bristol

Snow avalanches are potent natural hazards. They may flow at speeds of up to 50ms⁻¹ and endanger many inhabited regions. Large obstacles are sometimes built in the likely flow path of avalanches to protect villages by deflecting or arresting the oncoming flow. In this paper, laboratory investigations of rapid granular flows will be reported and models of their interaction with stationary obstacles will be developed. Specifically we will report how rapid flows may become airborne to form a coherent granular jet and how the flow may be deflected and undergo abrupt transitions (shocks) between flowing states.

“Reaction-diffusion models for wormlike micellar fluids”

A. Belmonte and Nestor Handzy,
Pennsylvania State University

The rheology and hydrodynamics of wormlike micellar fluids are known experimentally to include many novel instabilities, including shear banding and shear thickening, nontransient oscillations of rising bubbles and falling spheres, pendant drop stall and filament rupture. The differences with polymer fluid flows are usually attributed to the breaking and rejoining of the micellar tubes. We present a simple chemical reaction model for the constituents of wormlike micellar fluids, which reproduce many of the observed rheological features. We also discuss the mathematical implications of the physical existence of memory in these dynamic micelle populations.

“Propagation of the interface in a fluid suspension after the onset of shear flow”

C. Nouar, P. Riha and A. Lefevre

A measurement is carried out to determine the propagation of interface after the onset of shear flow between two coaxial cylinders in the fluid suspension exhibiting yield stress. The propagation of interface between the liquid-like and solid-like zones is evaluated from the velocity distribution in the former and the deformation of the latter zone. The deformation is visualised by the tracer spread on the free surface and the velocity distribution is measured by the ultrasound technique. The experimental interface position is related to the strain bound up with the end of plastic deformation and outset of viscous deformation in the solid-like zone. The origin of two sections separated by the interface depends on the proportion of acting hydrodynamic forces and inter-particle attractive forces. When the hydrodynamic disturbances predominate the attractive forces between particles the fluid suspension flows and vice versa. The complexity of inter-particle interactions in dense fluid suspensions limits the application of the concepts of classical mechanics to find a relation between individual particles dynamics and the macroscopic response to that. For this reason, the alternative stochastic scheme is proposed to specify the particle reaction to the start-up of flow between two coaxial cylinders.

“Flow of viscoplastic liquids through axisymmetric expansions-contractions”

M. Naccache,

Pontifícia Universidade Católica, Rio de Janeiro, Brazil

The flow of a viscoplastic fluid through an abrupt axisymmetric expansion followed by a contraction is analyzed numerically. The solution is obtained by solving the conservation equations of mass and momentum, via the finite volume method. In order to model the viscoplastic behavior of the liquid, the Generalized Newtonian Liquid constitutive equation was employed, in conjunction with a recently proposed viscosity function (Souza Mendes and Dutra, 2005). The numerical results are obtained for steady, inertialess flow. Velocity and stress fields are obtained for different combinations of geometric and rheological parameters. Yielded and unyielded regions are determined for these conditions. The results show a strong change of flow pattern at a critical value of the length-to-diameter ratio, and an apparent discontinuity of the velocity field in some cases, where the viscoplastic liquid seems to “fracture” near the core region of the flow.

“Viscoplasticity and geophysical flows: from field evidence to laboratory experiments”

C. Ancey,

Ecole Polytechnique Federale de Lausanne

Natural gravity-driven flows such as snow avalanches and debris flows are frequent phenomena in the Alps and other mountain ranges. These flows of bulk materials threaten man's activities and life; as a consequence, there is a rising demand for higher safety measures, which has given impetus to the development of models for predetermining the flow features and the probability of their occurrence. A key point in the development of these models is the constitutive equation.

Very early in the development of flow models for geophysical flows has appeared the notion of plastic behavior to describe the solid/fluid properties that a number of natural materials exhibit. Today, after several decades of investigations, the debate around the existence and nature of plastic yielding is still vivid. Among others, there are a number of open questions such as: after yielding, does the bulk behave like a frictional or a viscous material? Does plastic behavior properly describe the physical behavior? If so, how to measure the constitutive parameters? If not, does it mean that plasticity and related concepts only provide a conceptual representation without any clear physical support? In that case, is it licit to adjust the free parameters used in the plastic model from field data? Is there really a connection between granular-suspension flows at the natural and lab scales?

To tackle these questions, the talk is divided into two parts. First, we will focus on field evidence showing the existence of a plastic behavior in the case of snow avalanches. We will show that on most occasions, a simple Coulomb frictional model is sufficient to describe the main traits of an avalanche, but there is little hope that this model genuinely represents the physical processes governing the avalanche motion. Indeed, using Bayesian inference techniques, we have found that the variations in the friction coefficient (for different avalanches and paths) are not identically distributed from the statistical standpoint, in part because a number of parameters such as avalanche volume affects the value of the friction coefficient. Second, to better understand the rheological behavior of natural materials, we will consider laboratory experiments made on model suspensions (i.e., mixtures of beads, fine particles, and Newtonian interstitial fluids). Lab experiments reveal that a wide range of behavior can be exhibited depending on the solid concentration and shear rate.

Minute changes in the solid concentration in the fine/coarse particle content may significantly modify the rheological response of the material; for instance, frictional plastic and viscoplastic-like behaviors are observed for relatively similar suspensions in terms of solid concentration. In addition, in many cases, there is a complicated time evolution of the rheological response, starting from a power-law behavior at short times and evolving towards a purely frictional Coulomb behavior or a pseudo-Newtonian behavior at long times. Although a complete theoretical treatment of concentrated fine/coarse particle suspensions seems to be beyond our current capacity, simple dimensional arguments can be used to discriminate the expected rheological behaviors depending on the flow features and material characteristics.

“Long time extrusions of shallow viscoplastic fluids”

R. Sassi,
Università di Milano

Viscoplastic flows upon slopes are a possible idealization of many flows in several geophysical situations where yield stress is thought to play a role. If the fluid is relatively shallow and spreads slowly, asymptotic approximations can be used to build reduced models for the spreading dynamics (lubrication theory). We developed an ADI (Alternating Direction Implicit) numerical scheme to access reliably the large-time solutions of the model (fluid thickness). The numerical solutions were compared with the temporal power-law scalings expected from the similarity solutions.

“Multiple solutions of steady granular chute flows”

A. J. Hogg and Mark J. Woodhouse
University of Bristol

Highly agitated flows of particles down a rough inclined plane are modeled using kinetic theory, developed for granular materials. We calculate numerically fully developed flow profiles for the volume fraction, velocity and granular temperature of the flowing grains using a pseudospectral method and parametric continuation. We find regimes in which multiple solutions occur for a given volume flux of granular material and for these situations we assess the stability of the solutions.

“Conditions for static bubbles in viscoplastic fluids”

N. Dubash,
Imperial College

The slow motion of a gas bubble in a cylindrical column filled with a viscoplastic fluid is considered. Because of the yield stress of the fluid it is possible to have bubbles that do not rise. We adapt two of Prager's variational principles to our problem. From these, general stopping conditions are developed. We illustrate these conditions using specific bubble shapes e.g., spherical bubbles, axisymmetric bubbles, and compare the conditions against some basic experiments.

“Rheology and fluid mechanics of two-phases food fluids within circular pipes. Part A: General properties and rheological flow curves”

P. Perona,
Swiss Federal Institute of Technology

Many industrial food products show a non-Newtonian rheological behaviour of the shear thinning type with the presence of yield stress. From an engineering viewpoint, this latter is a characteristic of several food fluids. Among them can be included those products showing a high content of fibers as, for instance, fruits and vegetables. Fruits, for instance, have an internal structure typically associable to an agglomerate of un-coiled filament-like particles (apple puree) or to an ensemble of irregular spherical-like particles (peach puree) or both ones (pear and apricot purees) scattered within a main heterogeneous liquid matrix. Such complex structure is responsible for the Non-Newtonian behaviour. The rheological characterization for such products is therefore a fundamental step which importance ranges from speculative to more practical interests. For example, fluid characterization is relevant to the knowledge of the fluid dynamics behaviour and the heat transfer processes in pipe flow, which are fundamental to either improve or develop new industrial techniques. Some experimental results concerning with the static rheological characterization of several kinds of fruit purees (apple, apricot, peach and pear purees) at various dilution rate (i.e., percentage in volume of water) and temperature are presented. All the experiments were carried out using a pilot plant which functioning is based on the rectilinear pipe viscometer. The occurrence of either temporal or other anomalous effects that might influencing the results were controlled by doing the rheological characterization on two different pipe diameters. Time dependency was found to be relevant only in cases that implied a pronounced material degradation. The variability of viscosity on shear was instead always relevant and strongly dependent on both temperature and dilution degree (percentage in volume of water). In particular, the rheological flow curve was described by means of the Herschel-Bulkley model, which turns into the Ostwald and de Waele one for vanishing yield stress (i.e., high dilution rates). The magnitude of the yield for some sample was also reconstructed by processing the flow loop data with a method based on Tikhonov regularization.

Yeow, Y. L., Perona, P. & Leong, Y. K. “A reliable method of extracting the rheological properties of fruit purees from flow loop data.” *Journal of Food Science*, 67 (4), 2002, 1407-1411;

Perona, P. “Bostwick degree and rheological properties: an up-to-date viewpoint.” *Applied Rheology*, to appear.

“Rheology and fluid mechanics of two-phases food fluids within circular pipes. Part B: Laminar-turbulent transition and related resistance laws”

P. Perona,

Swiss Federal Institute of Technology

Some experimental results aiming at studying the laminar and incipient turbulent motion conditions for pipe flow dynamics of the food products encountered in Part A are presented. These results show that the influence of the dispersed solid phase (i.e., the woody fibers) on the resulting dynamics is different from one product to another. Resistance laws were obtained by plotting the classic Fanning friction factor (f) versus the generalized Reynolds number (Reg), this latter being calculated accordingly to the Metzner and Reed definition. Thus, the laminar motion conditions scaled well over the classical law valid also for Newtonian fluids, and proved once again its universal character if associated to a proper definition of the Reynolds number. As the fresh product generally has a high value of apparent viscosity and consistency, the experimental detection of the transition-to-turbulence was only possible above a certain dilution degree or above a given temperature level. The condition of transition to turbulence plotted on the plane $\{\tau_w, 8U/D\}$ (τ_w is the wall shear stress; U the mean velocity and D the pipe diameter) then moves along a curve toward the corresponding limiting value that is generally valid for water. As expected, on the $\{f, Reg\}$ logarithmic plane these curves show that the transition point for many fluids depends on the flow index of the rheological law, although in average such a point is located around a generalized Reynolds number of about 2100. Furthermore, a marked drag reduction effect was also detected for those fruits having a typical hair-like structure of the internal fibers. Since some of these products (eg. apple puree) have an internal structure that is quite sensitive to mechanical stresses, this study is useful to predict an upper limit for the flow rate. To this end, some tentative experiments aimed at detecting the breakdown mechanism occurring on both laminar and turbulent motion conditions were also performed. Both a qualitative (i.e., Optical Microscopy) and quantitative (changes in the rheological parameters) analysis showed that turbulent motion can potentially damaging the internal structure of such fluids, thus causing even marked changes of their both rheological and physicochemical properties.

Perona, P. “An experimental investigation of laminar-turbulent transition in complex fluids.” *Journal of Food Engineering*, 60 (2), 2003, 137-145;

Perona, P., Conti, R., Sordo, S. “Influence of turbulent motion on structural degradation of fruit purees.” *Journal of Food Engineering*, 52, 2002, 397-403;

“On the numerical simulation of visco-plastic flow, a la Bingham”

R. Glowinski,
University of Houston

The main goal of this lecture is to discuss the numerical simulation of visco-plastic flow, *a la Bingham*, including a history of the methodology to be described. A particular attention will be given to duality based methods whose main goal is to turn around the difficulties associated with the numerical treatment of the non-differentiable functional $\int |\mathbf{D}(\mathbf{v})| dx$, where \mathbf{v} is the flow velocity and $\mathbf{D}(\mathbf{v}) = \frac{1}{2}(\nabla \mathbf{v} + (\nabla \mathbf{v})^T)$. The techniques to be discussed will include: Augmented Lagrangians, time discretization by operator-splitting, efficient Stokes solvers to treat incompressibility, finite element approximations, Uzawa algorithms, etc... The results of numerical experiments will be used to validate the methodology under consideration

“Rayleigh-Benard stability of a Bingham fluid - some initial results”

I.A. Frigaard, D. Vola & J. Zhang

We present results on Rayleigh-Benard stability of a Bingham fluid. These flows are of relevance to many different applications: magma flows, switching devices for ER/MR fluids, cooking of chocolate, etc., as well as of fundamental interest. Our principal results are as follows. (i) For any positive yield stress the flow is linearly stable for all Rayleigh numbers, Ra . (ii) The flow is globally stable below a critical Rayleigh number, Ra_E , which is also the energy limit of the analogous Newtonian problem. (iii) For Rayleigh numbers that exceed Ra_E , we have conditional nonlinear stability, dependent on the size of the initial perturbations. (iv) The Prandtl number affects only the conditional stability bounds above Ra_E , and also the decay rates of the perturbation. (v) For all stable parameters we demonstrate that the velocity perturbation decays to zero in a finite time, (the timescale of which we are able to estimate), whereas the temperature perturbation decays exponentially. A series of computational results are presented that largely confirm the stability analysis. The computations are made using a numerical method based on the decomposition-coordination method of Fortin and Glowinski, with a suitable "cocktail" of numerical ingredients (low-order finite elements, velocity-pressure stabilisation, fully decoupled Uzawa-like algorithm). This solves the coupled thermo-fluid problem without need for regularization, which is necessary if one wishes to correctly model stopping of the flow. For sub-critical Rayleigh numbers, we observe that the fluid come to rest in a finite time by growth of the unyielded zones. The norm of the temperature decays exponentially, but we can observe two distinct decay timescales, depending on whether the fluid has stopped or not. For super-critical Rayleigh numbers we study the cellular flow patterns and attempt to use the computations to define a sharper conditional bound on the Bingham number

“Rayleigh-Benard-Poiseuille instability of Bingham fluid”

C. Metivier, C. Nouar, and J.P. Brancher
LEMTA, Nancy

The present communication deals with the linear and weakly nonlinear stability of the Rayleigh-Benard system with Poiseuille through-flow of Bingham fluid. In the linear analysis, we focus particularly on the situation where the yield stress approaches zero. By comparison with the Newtonian fluid, a discontinuous behaviour is observed and explained. The analysis of the stresses inside the plug zone, reveals that the dimensionless size of the perturbation has to satisfy an inequality involving the Bingham number. In the weakly nonlinear study, new results concerning the evolution of the plug zone are obtained.

“Viscoplastic, surface-tension-driven fingering”

Neil Balmforth, S. Ghadge, University of British Columbia and T. Myers, University of Cape Town

We consider the effect of a yield stress on the fingering instability of a contact line driven by surface tension. Theoretically, we develop a lubrication model to analyse the linear stability of a uniformly moving planar contact line. The results indicate that the introduction of the yield stress turns off the instability, This observation is consistent with some experimental results previously reported by de Bruyn et al, and by some qualitative experiments we conducted with kaolin suspensions.

“Roll waves in Mud”

N. Balmforth, University of British Columbia & J.J. Liu

The stability of a viscoplastic fluid film falling down an inclined plane is explored, with the aim of determining the critical Reynolds number for the onset of roll waves. The Herschel-Bulkley constitutive law is adopted and the fluid is assumed two-dimensional and incompressible. The linear stability problem is described for an equilibrium in the form of a uniform sheet flow, when perturbed by introducing an infinitesimal stress perturbation. This flow is stable for very high Reynolds numbers because the rigid plug riding atop the fluid layer cannot be deformed and the free surface remains flat. If the flow is perturbed by allowing arbitrarily small strain rates, on the other hand, the plug is immediately replaced by a weakly yielded “pseudo-plug” that can deform and reshape the free surface. This situation is modelled by lubrication theory at zero Reynolds number, and it is shown how the fluid exhibits free-surface instabilities at order-one Reynolds numbers. Simpler models based on vertical averages of the fluid equations are evaluated, and one particular model is identified that correctly predicts the onset of instability. That model is used to describe nonlinear roll waves.

Workshop participants (Page 1 of 2)

Alexandrou, Andreas	University of Cyprus	andalex@ucy.ac.cy
Ancey, Christophe	Ecole Polytechnique Federale de Lausanne	christophe.ancey@epfl.ch
Balmforth, Neil	University of British Columbia	njb@math.ubc.ca
Barnes, Howard	University of Wales, Aberystwyth	hab@aber.ac.uk
Belmonte, Andrew	Pennsylvania State University	belmonte@math.psu.edu
Bildhauer, Michael	Saarland University	bibi@math.uni-sb.de
Bonn, Daniel	ENS Paris/WZI Amsterdam	Daniel.Bonn@lps.ens.fr
Craster, Richard	Imperial College	r.craster@imperial.ac.uk
de Bruyn, John	University of Western Ontario	debruyn@uwo.ca
de Souza Mendes, Paulo	Pontifícia Universidade Católica - RJ	pmendes@mec.puc-rio.br
Dubash, Neville	Imperial College	neville.dubash@imperial.ac.uk
Entov, Vladimir	Institute for Problems in Mechanics, Russian Academy of Science, Moscow	entov@ipmnet.ru
Frigaard, Ian	University of British Columbia	frigaard@math.ubc.ca
Friskén, Barbara	Simon Fraser University	friskén@sfu.ca
Georgiou, Georgios	University of Cyprus	georgios@ucy.ac.cy
Ghadge, Shilpa	University of British Columbia	shilpa@math.ubc.ca
Glowinski, Roland	University of Houston	roland@math.uh.edu
Heuzey, Marie-Claude	Ecole Polytechnique	marie-claude.heuzey@polymtl.ca
Hogg, Andrew	University of Bristol	a.j.hogg@bris.ac.uk
Huilgol, Raj	Flinders University of South Australia	raj@infoeng.flinders.edu.au
Ionescu, Ioan	CNRS and University of Savoy	ionescu@univ-savoie.fr
Metivier, Christel	Institut National Polytechnique de Lorraine (INPL Nancy)	christel.metivier@ensem.inpl- nancy.fr
Mitsoulis, Evan	National Technical University of Athens (NTUA)	mitsouli@metal.ntua.gr
Møller, Peder	Ecole Normale Supérieure	peder.moller@gmail.com
Naccache, Monica	Pontifícia Universidade Católica – RJ	naccache@mec.puc-rio.br
Nouar, Cherif	CNRS-LEMETA Nancy	Cherif.Nouar@ensem.inpl-nancy.fr
Patterson, Mike	University of Cambridge	m.d.patterson@damtp.cam.ac.uk

Workshop participants (Page 2 of 2)

Perona, Paolo	Swiss Federal Institute of Technology	perona@ihw.baug.ethz.ch
Peysson, Yannick	French Institute for Petroleum (IFP)	yannick.peysson@ifp.fr
Pouliquen, Olivier	CNRS-University of Provence, Marseille	olivier.pouliquen@polytech.univ-mrs.fr
Putz, Andreas	University of British Columbia	putza@math.ubc.ca
Sassi, Roberto	Università di Milano (Italy)	Roberto.Sassi@unimi.it
Shelukhin, Vladimir	LIH/ Novosibirsk	shelukhin@hydro.nsc.ru
Sherwood, John	Schlumberger Cambridge Research	sherwood@cambridge.oilfield.slb.com
Slim, Anja	University of Cambridge	A.C.Slim@damtp.cam.ac.uk
Tsamopoulos, John	University of Patras	tsamo@chemeng.upatras.gr
Vola, Didier	IRSN France	didier.vola@irsn.fr
Wachs, Anthony	Institut Français du Pétrole, Paris	Anthony.WACHS@ifp.fr
Wilson, Stephen	University of Strathclyde	skwilson@strath.ac.uk