Mathematical Image Analysis and Processing

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1 Scientific Statement of the Workshop

Thanks to technological breakthroughs in the past few decades, mankind is now able to see images of worlds without (distant planets, galaxies, and the surface terrain of the Earth) and worlds within (human organs, geological imaging, and atomic and molecular structures at the nanoscale level). See Figure 1.

As the state-of-the-art imaging technologies became more and more advanced, yielding scientific data at unprecedented detail and volume, the need to process and interpret all the data has made image processing and computer vision also increasingly important. Sources of data that have to be routinely dealt with today include video transmission, wireless communication, automatic fingerprint processing, massive databanks, non-weary and accurate automatic airport screening (e.g., the USA Federal government's experiment of retina-image based automatic screening at the Minneapolis-St. Paul International Airport), robust night vision for civilian rescue workers or battlefield soldiers, and vision repair for patients with vision defects.

Input from mathematicians has had a fundamental impact on many scientific disciplines. When accurate, robust, stable, and efficient tools were required in more traditional areas of science and technology, mathematics often played a role in helping to supply them. No doubt the same will be true in the case of imaging and vision sciences.

Figure 1: (a) Images from Mars' explorer, (b) Brain MRI image.

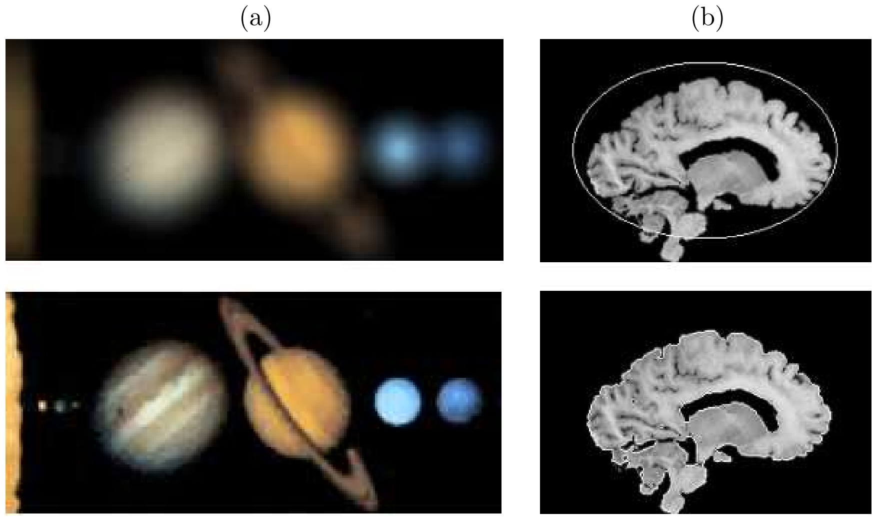

Figure 2: Example of image restoration (a) deblurring, and (b) image segmentation

The workshop *Mathematical Image Analysis and Processing* was motivated by both the imminence of vision sciences, and this principle about the role of mathematics. With the support of BIRS, it provided a solid platform for enthusiastic mathematicians to further their research in imaging and vision sciences, and start collaborations that will lead to new discoveries.

2 Mathematical Statement of the Workshop

Mathematical Image Processing is a rapidly growing field. As such, there are many different approaches for addressing similar questions. The main task of our workshop has been to concentrate on a few mathematically intriguing problems, and to allow researchers to present the state-of-the-art in their approaches to these problems. In this way, the workshop has achieved two goals. First, it has provided a forum where results from different approaches can be systematically compared as well as integrated. Second, by making sure a variety of mathematical areas (from pure theoretical analysis to practical computational techniques) are represented, the workshop will encourage more mathematicians to work on imaging and vision problems.

The scientific structure of the workshop was based upon the following intrinsic principle: *from image analysis to image processing*. Designing a successful processing technique relies on having a successful model for images themselves. Image analysis mainly focuses on image spaces and efficient ways to represent images, such as spectral analysis, wavelets, statistics, level-sets and PDEs. Image processing involves modifying the original images in order to improve the visual qualities or to extract valuable information for further higher-level processing. Some familiar processing tasks are image restoration, compression, segmentation, shape analysis, and texture extraction (see Figure 2).

The three crucial ingredients of image analysis and processing are: modeling, analysis, and computational implementation. In the past couple of decades, mathematicians have been able to make substantial contributions in all these areas. Our workshop was instrumental in helping both active and newly interested mathematicians to refine and further existing ideas, as well as to highlight and concentrate on challenging problems.

3 Participants

The glowing reputation of BIRS allowed us to attract a highly diverse group of participants for our workshop.

Figure 3: Some of our participants.

Disciplinary Diversity. The themes and missions of the workshop successfully attracted not only mathematicians, but also scientists from closely connected areas of electrical engineering, computer science, information sciences, biomedical engineering, industrial engineering, psychology, as well as from national research labs. Such multidisciplinary interaction and collaboration is a characteristic of mathematical image analysis and processing as a growing field in applied mathematics, and is a solid foundation for long-term healthy development.

Global Diversity. The majority of the thirty-eight participants came from Canada, Mexico, and the USA. In addition, there was representation from Austria, Finland, France, Germany, Israel, Italy, and Norway. Such global collaboration will make it easier for researchers to communicate, coordinate, and optimize their scientific and academic resources (e.g., funding or holding future conferences on mathematical image processing, training and exchanging research assistants or postdocs, and collaborating on and unifying research projects).

Visible Minorities. There were eight female participants and eleven participants from racial groups that are under-represented in North American science.

Level Diversity. There was a good mixture of participants from all levels: graduate students, post-doctoral researchers, tenure-track professors, and senior scientists. Senior scientists who participated are: Bill Allard, Andrea Bertozzi, Tony Chan, Gerardo E. García Almeida, Jim Little, Brad Lucier, Riccardo March, Dimitris Metaxas, Mila Nikolova, John Oliensis, Mary Pugh, Martin Rumpf, Fadil Santosa, Otmar Scherzer, Volker Schmidt, Jayant Shah, Kaleem Siddiqi, Xue-Chang Tai, Baba Vemuri, Luminita Vese, Curt Vogel, and Ross Whitaker. Post-docs and tenure-track assistant professors who participated are: Mimi Boutin, Selim Esedoglu, John Greer, Sinan Gunturk, Sung-Ha Kang, Stacey Levine, Kirsi Majava, Francois Malgouyres, Jackie Shen, Richard Tsai, Kevin Vixie, Lior Wolf, and Haomin Zhou. There were three advanced graduate students present: Toni Buades-Capo, Fred Park and Alon Spira. This range of levels demonstrates the strength of mathematical image analysis and processing in both contemporary scientific arenas and its potential in future ones.

4 Scientific Overview of the Workshop:

The workshop covered a variety of topics and methodologies in contemporary image and vision analysis.
In terms of scientific areas, the workshop touched upon:

(a) computer vision, especially shape analysis and scene reconstruction;

(b) theoretical image analysis;

(c) general image processing including edge detection, denoising, deblurring, inpainting, registration, and segmentation;

(d) biomedical image processing including diffusion tensor imaging, tumor detection, and retinal movement quantification.

(e) industrial image processing, including the automobile and printer industries.

(f) information and communication sciences, including efficient image data compression, coding, and error concealment.

The following mathematical methodologies were represented at the workshop:

(1) variational optimization, especially on non-quadratic, non-convex, geometry-oriented, and data adapted "energy" functionals;

(2) inverse problems for ill-posed problems and regularization;

(3) nonlinear partial differential equations (PDE), to model, simulate, or achieve equivalent physical actions such as diffusion, convection, free-boundary interface motions, and phase transitions;

(4) differential geometry, for processing data information on general Riemannian manifolds and embedded surfaces, as well as for processing generic geometric information such as total curvatures and principle curvatures;

(5) Lie groups and invariant theory, for studying affine or projective invariants of image acquisition from camera motions and different views;

(6) quantum information theory, especially using the quantum probabilistic view to develop novel approaches to mathematical learning theory;

(7) statistical and information theory, to process data features or patterns that are beyond the effective description of deterministic models such as PDEs and variational energies;

(8) harmonic analysis, on analyzing wavelets compressing and coding schemes, as well as investigating the interaction between such atomic decompositions and variational/PDE methods;

(9) real and functional analysis, such as using distributions and oscillatory functional spaces (e.g., BMO) to model textures;

(10) computational logic, e.g., how to make sound Boolean decisions for noisy or multi-channel data;

(11) numerical analysis, including the stability and accuracy of high-order nonlinear PDE schemes;

(12) scientific computation, including the level-set method, numerical PDE, Γ-convergence regularization, thresholding dynamics, iterative algorithms, as well as multiphase computation.

The applications of theories and methodologies of the workshop have been found in:

(i) medicine and the health sciences, including tumor detection and robust nerve fiber tracing in the brain;

(ii) industrial engineering, such as designing an automatic feed-back control (vision) system in the automobile industry and improving the quality of inkjet printers;

(iii) astronomy, such as enhancing and improving the quality of telescope observations by adaptive denoising, deblurring, and repairing;

(iv) communication technologies, including efficient data coding and error concealment for noisy or lossy channels;

(v) artificial intelligence, including parameter-free learning processes;

(vi) movie and art restoration, and computer graphics;

(a) (b)

Figure 4: (a) Dr. Martin Rumpf and (b) Dr. Tony Chan giving talks at the workshop.

(vii) surveillance video and airport security;

(viii) robot vision system, including object and scene perception;

(ix) military applications, such as for autopiloting planes to automatically track enemy vehicles and moving information.

Such a broad scope and range made Mathematical Image Analysis and Processing'04 a unique workshop. The participants found themselves in an ideal environment where they could freely communicate their research ideas as well as be nourished with fresh ideas from other participants.

5 Highlights of Presentations

The following are not merely summaries of the presentations, but also highlight how the researchers have made novel contributions as well as what new important trends they represent. (The summaries are given in presentation order.)

Dr. Baba Vemuri, from the University of Florida, proposed novel metrics for measuring the distances between tensors, as motivated by the application of diffusion tensor imaging in medical MRI. The talk showed the power of combining knowledge and tools from different areas, such as the probabilistic view of diffusion tensors (e.g., Gaussian distributions, the Kullback-Leibler distances between distributions, etc.), variational/PDE methods like region based active contour models, and restoration of Riemannian features (e.g., unit vectors on the spheres or orthonormal frames). The talk also demonstrated the growing realization that any imaging model or computational scheme often crucially depends upon using the proper metrics or measures.

Dr. Stacey Levine, from Duquesne University, presented her work on applying non-standard growth functionals for image denoising and image decomposition. Inspired by the previous works of Chambolle and Lions (1997), You and Kaveh (2000), and Chan et al. (1998), Dr. Levine studied the following functional on a given image domain Ω:

$$\int_\Omega \phi(x, \nabla u) dx,$$

where, unlike the conventional uniform Sobolev norm $\phi(x, \mathbf{p}) = |\mathbf{p}|^2$ or the Total Variation Radon measure $\phi(x, \mathbf{p}) = |\mathbf{p}|$, a non-standard growth exponent $q(x)$ is incorporated by

$$\phi(x, \mathbf{p}) = \frac{1}{q(x)} |\mathbf{p}|^{q(x)}, \quad |\mathbf{p}| < 1; \qquad |\mathbf{p}| - 1 + \frac{1}{q(x)}, \quad \text{otherwise.}$$

The spatially dependent growth exponent $q(x)$ is assumed between $1+\epsilon$ and 2 in order to ensure convexity. Dr. Levine studied both the mathematical theory (e.g., existence and uniqueness) and the computational performance of such models. The experiments showed noticeable improvement over conventional models for denoising and texture extraction.

Dr. Otmar Scherzer, from the University of Innsbruck, talked about studying illposed inverse problems using general nonlinear, non-differentiable, or non-convex regularizers. One particular problem is to restore an image whose pixels have been randomly (and blindly) switched. When the random displacement is local, a Taylor expansion inspired the following degenerate, nonlinear, and non-convex functional for solving such an ill-posed inverse problem:

$$\min_u \int_\Omega |Du| + \int_\Omega \lambda(x, u, \nabla u)(u - u_0)^2 dx,$$

where $\lambda(x, u, \nabla u) = \lambda + |\nabla u|^{-2}$, with the constant λ also handling potential intensity noises. The non-convex and singular dependence of $\lambda(x, u, \nabla u)$ on ∇u leads to analytical difficulties. Dr. Scherzer showed how to resolve these issues.

Alon Spira, from the Technion, discussed a general framework for evolving images and curves on parametrized Riemannian manifolds. Of particular interest for image processing and vision tasks are the Eikonal equations and Beltrami flows on Riemannian manifolds. The computational efforts presented in the work are representative of the literature for robustly and efficiently handling geometry while performing certain image processing tasks.

Dr. Martin Rumpf, from the Gerhard Mercator Universität-Gesamthochschule, focused on his work on high-order geometric flows for anisotropic images and surface processing, as motivated by Willmore-type energies and anisotropic surface functionals, e.g.,

$$A_\gamma(M) = \int_M \gamma(n) da,$$

where M denotes an embedded surface in $\mathbb{R}^3$, n its unit normal, and da the surface element. The anisotropy comes from the choice of γ. If γ has the general form of $\gamma(x, n, H, K)$ with mean curvature H and Gaussian curvature K, then the resulting Euler-Lagrange equations are inevitably of high order (e.g., 4th order). Dr. Rumpf also presented efficient numerical methods for handling such high order geometric flows.

Dr. Lior Wolf, from MIT's Center for Biological and Computational Learning, presented his recent work on a novel learning theory inspired by the Born Rule from Quantum Mechanics. Let Ψ be a pure quantum state (represented by a column vector for simplicity) and $\rho = \Psi\Psi^T$ be its associated projection. Let M be a rank-one observable, i.e., $M = aa^T$ for some column vector a. Then the Born Rule claims that the expected value of the observable M is given by

$$\text{trace}(M\rho) = \Psi^T M \Psi = |\langle a, \Psi\rangle|^2.$$

Dr. Wolf explained how one can design novel learning algorithms based on the Born Rule, especially for data clustering (either two-class or multiple-class).

Dr. Mila Nikolova, from the Ecole Normale Superieure de Cachan, presented her work on edge recovery via nonconvex regularized least-squares. The properties of restored images and signals were carefully investigated based on the shape information of the regularization function. One remarkable result revealed a major difference between the edge-preserving convex functionals and non-convex regularizers: in the latter case, given a (local) minimizer the (intensity) difference between adjacent pixels are either suppressed or enhanced.

Toni Buades-Capo, from the Ecole Normale Superieure de Cachan, presented his work on rigorously comparing the performance of different image denoising algorithms and models. Work of this nature is of paramount importance to the imaging and vision literature, since for any given task, different research groups often develop different models and algorithms. Image denoising is a well-known example. Popular denoising methods include spectral methods, Wiener filtering, wavelet thresholding, adaptive filtering, anisotropic diffusions, etc. The key issue in developing rigorous comparative criteria is how to properly define performance measures, and how to properly interpret them from both vision and information theoretic points of view. That is; how to move beyond the "eyeball metric" in which the researcher visually judges the quality of the method, risking the possibility of bias.

Dr. Brad Lucier, from Purdue, investigated the variational smoothing of a class of Besov images $B^1_\infty(L^1)$ based on both a wavelet representation and on direct pixel-domain smoothing. This work reflects a growing need and trend for hybridizing different methodologies in image processing. Lucier's earlier works with Chambolle and DeVore in 1998 initiated the interaction between the variational formulations and wavelet algorithms. The present work further expands this direction by comparing the performance using different (but equivalent) formulations of the Besov semi-norms.

Dr. Ross Whitaker, from the University of Utah, presented impressive computational results for image denoising and other applications, based on an information-theoretic approach for adaptive filtering. The results appear superior to those in the variational literature which often implicitly assume that the underlying ideal images belong to the space of functions of bounded variation, $\mathrm{BV}(\Omega)$, or to some Sobolev space $W^{\alpha,p}(\Omega)$. The information-theoretic and statistical nature of Dr. Whitaker's new approach enables more robust and general prediction of individual pixel values based on local information as well as on suitable entropy measures. The work reflected the benefit of applying information-theoretic tools (often developed for the information and communication sciences) to image and vision analysis.

Dr. Luminita Vese, from UCLA, explored tools from real and functional analysis to study image decomposition. The primary goal of image decomposition is to separate a given image u_0 into visually or functionally meaningful components. A popular framework, initiated by Yves Meyer, is to decompose u_0 into $u + v + w$, where $u \in \mathrm{BV}$, w is Gaussian white noise, and v is an oscillatory pattern in $\mathrm{div}(L^\infty)$. The goal of solving such a model is to perform three functions at once — to denoise the image (find w), to segment the image (find u), and to find the textures of the various surfaces (find v). Dr. Vese presented her work on how to compute her new model using div(BMO) to model v, where BMO denotes all the locally summable functions with bounded mean oscillations.

Dr. Xue-Cheng Tai, from the University of Bergen, discussed how to develop computational and numerical analysis on piecewise constant level-set methods. Unlike the level-set setting of Osher and Sethian (1987) for which the level-set functions are Lipschitz continuous, Dr. Tai proposed employing piecewise constant level-set functions. The range of the level set function is preassigned, e.g., a finite set of integers, or merely binary values of -1 and $+1$. Dr. Tai also presented the successful applications of his methods to digital image processing as well as to solving ill-posed inverse problems.

Dr. Bill Allard from Duke presented his new results on the regularity of level sets of minimizers to total variation based image denoising models. These results, which draw on tools from geometric measure theory, apply to a wide range of models that have been considered in the image processing literature. Dr. Allard's results thus elucidate common features of these models, especially in regard to the bias that these models introduce (because of their regularization terms) into image reconstructions. Dr. Allard's results also allowed him to construct relevant and nontrivial exact solutions to several of these total variation based models. Such exact solutions are valuable in that they allow us to rigorously contrast the behavior of competing variational models. They also constitute test cases for numerical algorithms, so that the algorithms' performance can be scientifically measured instead of judging them solely on whether their results are visually pleasing or not.

Dr. Kevin Vixie, from Los Alamos National Lab, discussed his work investigating exact solutions to a variational denoising model based on the TV regularizer with L^1 fidelity. When the given target image is an indicator function of some set, under certain conditions the exact solutions can be characterized in terms of disks with appropriate radii.

Dr. Jayant Shah, from Northeastern University, a pioneer in mathematical image and vision analysis, presented some new developments on processing the skeleton information of generic shapes. The skeleton of a given closed shape with Lipschitz boundary is the locus along which the distance function of shape boundary has first order singularities, like the ridge of a mountain range. In computer vision, skeleton information has been used for object recognition, detection, and differentiation. The advance in Dr. Shah's work is to study gray skeletons instead of conventional binary ones.

Dr. Riccardo March, from the Istituto per le Applicazioni del Calcolo, presented his work on the curvature-dependent nonlinear and nonconvex functionals emerging from recent works in image processing. In order to more faithfully process edge information in models, Dr. March proposed to incorporate the information about endpoints, length, and curvature into the variational formulation. A Γ-convergence theory is then developed for such geometric functionals. It allows approximating the original functional by ones that are numerically much more convenient. Dr. March then explained the main ideas of the proof that establishes the convergence of approximate functionals to the original one. The work presented was an example of how rigorous mathematics helped solve some very practical problems encountered in image processing and computer vision.

Fred Park, from UCLA, presented his work on a total variation based model for simultaneous image inpainting and blind deconvolution. Blind deconvolution is used for recovering information from telescope images such as that in Figure 2; image inpainting is how scratches are "removed" from old photographs. He demonstrated that these tasks are best treated as a coupled image processing task. This allows him to take boundary conditions for the deconvolution that are naturally generated by the inpainting; thus reducing ringing effects that can arise from a poor choice of boundary conditions.

Dr. Haomin Zhou, from Georgia Tech, spoke about his efforts to combine PDE ideas and methods with wavelet techniques. A new class of singularity-adapted and more efficient wavelet transforms are invented by using shock-capturing schemes from computational fluid dynamics. Dr. Zhou also discussed how to use variational approaches and well-chosen functionals in the pixel domain to help improve the quality of wavelet compression and interpolation. Such work represents the exciting and necessary trend of combining the variational/PDE methodology with harmonic analysis.

Dr. Richard Tsai, from the UT Austin, presented a fast algorithm for constructing minimizing sequences for the Mumford-Shah segmentation functional. Inspired by earlier works of Merriman, Bence, and Osher on threshold dynamics for geometric curve evolution, Dr. Tsai discussed how the new fast algorithm was discovered and should be understood, and presented numerical results illustrating its effectiveness.

Dr. Kaleem Siddiqi, from McGill University, discussed a novel approach to shape analysis. Given the distance function of a shape, one computes its gradient. Loosely speaking, Dr. Siddiqi then views this gradient as a flux, reducing the problem of finding the Blum skeleton of a shape to finding sources in the flux.

Dr. Curt Vogel, from Montana State University, presented a new image processing problem which is markedly different from most. His goal is to robustly estimate and track the motion of the retina, based on high resolution scan data produced by laser devices. Retinal tracking is a crucial step in understanding the early human visual system. The major challenge is to distinguish irrelevant eye motion from the intrinsic motion of the retina, and to properly determine the optical properties. This work signifies the importance of image and vision processing in biological data probing and analysis.

Dr. Tony Chan, from UCLA, presented a new framework of logical segmentation for multichannel images as well as logical tracking for video images. This work improves the applicability of the conventional Mumford-Shah segmentation model by incorporating Boolean algebra. The work is especially relevant when information from multiple sources (such as different color channels, or even different imaging modalities) needs to be combined for the segmentation task.

Dr. Dimitris Metaxas, from Rutgers University, presented work in deforming shapes and interior textures (Metamorphs) as applied to medical imaging of the human heart. He also presented work in computer animation of breaking water waves. A major advantage of his algorithms was their computational efficiency.

Dr. Kirsi Majava, from the University of Jyväskylä, discussed her work on applying the active-set algorithm for nonsmooth variational optimization problems like

$$\min_u \int_\Omega |u - u_0|^s dx + \beta \int_\Omega |\nabla u|^r dx,$$

commonly seen in image denoising and restoration. She presented computational results suggesting this new algorithm is very promising. Potential connections to discretized active contour algorithms were suggested by the audience and further investigation into this novel algorithm is a hot topic.

Dr. Francois Malgouyres, from the Universite Paris 13, presented his work on achieving good image compression schemes through projections on polyhedral sets, extending his earlier work on image restoration.

Dr. Fadil Santosa, from the University of Minnesota, presented an interesting inverse problem arising from the imaging of spotwelds. A typical car has more than 20,000 spotwelds holding together metal sheets. The evaluation and monitoring of their quality is achieved by noninvasive thermal imaging devices. Dr. Santosa has been able to develop models, analyze the resulting illposed inverse problems, as well as computationally simulate the models based on regularization techniques.

Dr. Sinan Gunturk, from New York University, explored the mathematics behind a well-known class of analog-to-digital converters in signal processing ($\Sigma\Delta$-transition) as well as a similar type of converter in modern inkjet printers, called digital halftoning algorithms. Dr. Gunturk also discussed how to use modern multiscale ideas to develop novel and more accurate halftoning algorithms.

Dr. Andrea Bertozzi from UCLA, and Dr. John Greer, from New York University, both presented interesting work applying high-order PDE to image denoising, regularization, and image inpainting. High order PDE have emerged in the recent literature of image processing, such as the inpainting model of Bertalmio et. al. (2000):

$$u_t + (\nabla u)^{\perp} \cdot \nabla(\Delta u) = 0$$

and the LCIS (low-curvature image simplifier) equation:

$$u_t + \nabla \cdot (g(|\Delta u|)\nabla \Delta u) = 0.$$

Rigorous study of the existence, uniqueness, and proper boundary conditions is highly challenging and is needed in the image processing literature. The works of Dr. Bertozzi and Dr. Greer shed fresh light on both the analytical and algorithmic structures on these equations.

Dr. Mimi Boutin, from Purdue, presented work on applying invariant theory and the moving-frame method to the reconstruction of 3-D scenes from 2-D image projections. These tools from Lie Groups can extract valuable information parameters while discarding superfluous unknowns, and thus lead to a simple and robust scene reconstruction theory.

Dr. Gerardo Garcia Almeida, from the University Antonoma de Yucatan, presented his work on integral equations of the first kind. He presented anisotropic generalizations of the Tikhonov regularization and then sought the optimal value for the regularization parameter. The choice of the image space is particularly important for this analysis. He considers anisotropic Nikol'ski-Besov space to give more possibilities for applications of this method. His methods are from harmonic analysis.

Dr. Volker Schmidt, from the Universität Ulm, presented a new method for the statistical analysis of binary features. By using tools such as convex and stochastic geometry, the morphological image characteristics are robustly estimated statistically. This was a lovely piece of work in which he was able to extract topological features of all degrees (points, edges, facets, etc.)