Correlation chains and quantum hierarchies

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1 Overview of the Field

The past decade has witnessed a burst of interactions between operator algebras and quantum information theory, some of the prime examples of which are the announced resolution in the negative [6] of the Connes Embedding Problem (CEP) in operator algebras and the Tsirelson Problem in theoretical physics, the refutal of the Strong Tsirelson Conjecture [11] and the answer to the Weak Tsirelson Problem [10]. These questions, open until recently, build on Bell's seminar work from 1964, establishing the distinction between classical and quantum correlation sets for two parties participating in a quantum experiment. Due to these recent efforts, it is now known that the classical model, the quantum (or tensor product) model, the approximately quantum model and the quantum commuting model of quantum mechanics give rise to a strictly increasing chain of correlation types.

Viewing classical no-signalling correlations as classical bipartite information channels, and generalising to quantum channels (where the domain and co-domain are tensor products of matrix algebras), two of the members of the team introduced [12] quantum no-signalling correlation types and established a similar inclusion chain on the quantum level. One of the motivations behind this was to study quantum resource theories in the context of non-local games – cooperative games, played by two spacially separated and non-communicating players against a verifier. The different no-signalling correlation types here are used as strategies of the players, each corresponding to a certain type of quantum resource they may possess. The strict inequalities between the game values (that is, the optimal winning probabilities), when different types of resources are available to the players, is at the heart of the work [6]. Values of games with quantum inputs and outputs have been considered in the past (e.g., [3, 7]), and it is of substantial interest to study non-local games whose inputs and/or outputs are states in the quantum commuting, or approximately quantum, models, referred to in the previous paragraph. Indeed, results from [7] were recently leveraged to give a simplified resolution of the weak Tsirelson conjecture [2].

2 Recent Progress

A crucial body of techniques employed in the groundbreaking work [6] is that of self-testing quantum systems. Roughly speaking, a bipartite conditional distribution (a.k.a. correlation) is a self-test if there are unique local measurements and a bipartite state (up to local equivalence) which realize the conditional distribution. Non-local games whose winning strategies are a self-test can therefore only be won in a unique fashion, providing a certificate for quantum systems.

Very recently, self-testing of classical no-signalling correlations in the tensor product model was characterised in operator algebraic terms [8]. The motivation being to open new pathways towards understanding the work of [6] in an operator algebraic context (given its refutation of CEP). This, together with related results on the weak and strong Tsirelson conjectures [2, 10, 11], warrants the study self-testing of quantum no-signalling correlations.

Progress in the described area will depend on the successful use of operator space methods and quantum game intuition, and on fluency with the quantum commuting model of quantum mechanics. Members of the team have pioneered both the commuting operator approach to quantum informational questions (see [4, 9]) and a formalised quantum non-local game theory (see [1, 12]). More recently, the team members completed a paper on quantum game values [5].

3 Scientific Progress Made

During the Research in Teams meeting, a formalism for defining and studying self-testing of quantum no-signalling correlations was established using the theory of operator systems. In that formalism, the tensor product model results of [8] were generalised to the quantum hierarchy, and examples of self-testing in that hierarchy were proposed.

A suitable notion of local isometry in the commuting operator framework was also established, together with a related local dilation order. These were used to characterise self-testing in the commuting operator model for both classical and quantum no-signalling correlations, simultaneously completing an avenue left open from [8], and providing a new approach to constructing infinite-dimensional self-tests in the commuting operator framework, which is an open problem.

4 Outcome of the Meeting

The results of the meeting will produce a novel addition to the growing literature on self-testing, opening many new avenues of scientific exploration at the interface of operator algebras, quantum entanglement, and quantum foundations. We anticipate having a paper ready for submission in early 2024.

References

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