BIRS 11w5085: Interactions between contact symplectic topology and gauge theory in dimensions 3 and 4

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1 Introduction

This workshop focussed on interactions between contact and symplectic geometry, gauge theory, and low-dimensional topology. Each of these subjects is an active area of current research and interactions between them have led to breakthroughs on long standing problems. Our workshop was a follow-up to the BIRS events *Interactions of geometry and topology in low dimensions* from March of 2007 and *Interactions of Geometry and Topology in dimensions 3 and 4* from March 2009. Because the fields are progressing at a rapid pace, there were many new and interesting results presented at the workshop and new projects were initiated at the workshop as well.

Participants were selected from among the world experts in these areas. The organizers made an effort to balance interest between the different research areas and to ensure that the most important current trends were well represented. There was a good mixture of well-established researchers (Honda, Lisca, Matic, Mrowka, Stern) and younger talented mathematicians (Hom, Lekili, Ma'u, Vela-Vick, Vertesi, Zarev). This stimulated many lively discussions and enabled a rich exchange of ideas in all directions.

2 Overview of the Field

Over the last several decades it has become clear that the topology of manifolds in low-dimensions is subtly and beautifully intertwined with diverse flavors of geometry, like hyperbolic, symplectic and contact, as well as ideas from physics, such as gauge theories and mirror symmetry. Collaborations among people working in these diverse areas has exploded over the last few years resulting in the solutions to venerable conjectures in topology as well as the birth of entire new sub-fields and perspectives in these areas. Highlights of some of the more spectacular recent results include the characterization of which 3–manifolds admit a symplectic structure when crossed with S^1, the Heegaard-Floer characterization of fibered knots, the proof of Property P for nontrivial knots in S^3, the solution to the Weinstein conjecture (and generalizations of it) and a deepening of our understanding of exotic smooth and symplectic structures on 4-manifolds. Critical tools in these developments are invariants inspired by gauge theories and topological quantum field theories. These invariants – Donaldson-Floer, Seiberg-Witten, Ozsváth-Szabó, Khovanov homology and Embedded contact homology to name a few – have intriguing relations among them, and a better understanding of these will lead to significant progress not only in topology but also in contact and symplectic geometry and physics. An even more promising direction is the interplay between these invariants and more constructive approaches to low-dimensional manifolds – open book decompositions of contact 3-manifolds, symplectic fillings, Lefschetz fibrations, knot surgery constructions among many others. This interaction between powerful invariants and constructive methods is more than ever one of the driving forces in this subject. Below we will survey some of the most active branches of low-dimensional topology, thereby outlining natural directions and objectives for the workshop.

Unification of invariants: Recently there has been much progress in showing various invariants defined in starkly different ways actually compute the same thing. This has allowed for many striking results. For example, as Taubes and Hutchings have made progress identifying Seiberg-Witten Floer theory with Embedded Contact Homology, Taubes has managed to spin these ideas into a proof of the much studied Weinstein Conjecture in dimension 3: for any compact oriented 3-manifold M and α a contact 1-form on M, the vector field that generates the kernel of the 2-form $d\alpha$ has at least one closed integral curve. Further developments have allowed for extensions and refinements of the Weinstein conjecture and it appears we are on the cusp of identifying the two theories. The ramifications of such a convergence of theories are as yet unknown but given the spectacular results following from progress on this program, one expects great things. For instance, progress on Pidstrigatch and Tyurin's program to prove the Witten conjecture relating instanton Floer homology with Seiberg-Witten Floer homology has led to the solution of the famous conjecture that all non trivial knots in S^3 have Property P: that is that non trivial surgery yields a manifold with non trivial fundamental group. Another exciting, spectacular, and very recent instance of unification of invariants is the work in progress of Kutluhan, Lee and Taubes relating Seiberg-Witten Floer homology and Heegaard-Floer homology, and that of Colin, Ghiggini and Honda relating Embedded Contact homology to Heegaard-Floer homology.

Another current trend in the area is the understanding of the relationship between the various invariants of Floer type for knots and 3-manifolds and Khovanov homology. Khovanov homology was constructed as a categorification of the Jones polynomial of knots and its nature is very algebraic and combinatorial. Ozsváth and Szabó derived a spectral sequence whose E^2 term is a suitable variant of Khovanov's homology for a link, converging to the Heegaard Floer homology of the double branched cover of the link. The progress accomplished on combinatorial Heegaard-Floer homology has already enabled Manolescu and Ozsváth to explore further the relationship between the two theories, through the notion of homological thinness. There are good reasons to believe that this will be an active area of research for the coming years, as this should also be related to the link invariant constructed by Seidel and Smith using the the symplectic geometry of nilpotent slices. In another direction, Kronheimer and Mrowka have established an intriguing relationship between the Khovanov (co)homology and the knot instanton Floer homology, again via a spectral sequence, and their new work builds on their foundational results on singular instanton connections over 4-manifolds and has application to answering affirmatively the question whether Khovanov homology detects the unknot. (The answer to the same question with the Jones polynomial is not known.)

Developing computation techniques: Most of the topological invariants arising from gauge theory and contact / symplectic topology rely extensively on analytical tools, which makes explicit computations particularly difficult since information about spaces of solutions to such PDE problems is scarce. In the past few years there has been dramatic progress in combinatorial approaches to Ozsváth-Szabó theory as well as Contact Homology. Indeed, the problem of combinatorially constructing Heegaard-Floer groups without resorting to counting pseudo-holomorphic curves has taken a very promising turn as knot Floer homology was given a purely combinatorial interpretation by Manolescu, Ozsváth and Sarkar. This has already led to progress in the classification of transverse knots in contact manifolds as well as work by Ng on bounds for the Thurston-Bennequin invariant of Legendrian knots. It is expected that the theory will progress greatly over the course of the next few years thanks to the combinatorial set-up. Moreover, Bourgeois, Ekholm and Eliashberg have constructed an exact sequence that allows one to compute the contact homology of a contact manifold obtained from "Legendrian surgery" on another one. This construction is particularly "simple" in dimension 3 where there is essentially an algorithm for writing down the contact homology of a contact 3-manifold obtained from Legendrian surgery on a Legendrian knot. With recent progress on the classification of Legendrian knots in various knots types this could yield a flood of information about contact 3-manifolds.

The recent work of Lipshitz, Ozsváth and Thurston has opened a whole new direction by extending Heegaard-Floer homology to the case of 3-manifolds with boundary. Among other applications, this allows one to compute Heegaard-Floer homology by decomposing a 3-manifold into a sequence of elementary cobordisms between oriented surfaces.

Exploiting interactions between constructions and invariants: The emergence of invariants of embeddings from contact homology is also one of the promising avenues of research in the area. Given a manifold embedded in Euclidean space, one can look at its unit conormal bundle in the unit cotangent bundle of Euclidean space to get a Legendrian submanifold. Computing the contact homology of this Legendrian gives an invariant of the original embedding. Ekholm, Etnyre, Ng and Sullivan have recently rigorously computed this invariant for knots in 3-space and shown it is equal to a very powerful combinatorial invariant defined by Ng. This invariant has surprising connections with many classical invariants of knots and seems quite strong. Exploring this new invariant of knots and extending it to other situations should be a fruitful line of research for years to come. Moreover, contact homology is only the tip of the iceberg of Symplectic Field Theory (SFT). This theory, introduced by Eliashberg, Givental and Hofer, has been an inspirational and driving force in symplectic geometry for over a decade now, and recent advances in its rigorous definition suggest that a precise formulation of the relative version should emerge in the coming years. It appears there will still be much work to do to extract computable and meaningful pieces that one can use in applications. In the end though, it is expected that the theory will be invaluable in symplectic and contact geometry and will provide more invariants, not only for Legendrian knots in contact 3-manifolds and Lagrangian cobordisms between them, but also for topological knots by considering the conormal construction mentioned above. Evidence for this comes from Abouzaid's recent demonstration that the symplectic geometry of cotangent bundles can be used to distinguish exotic smooth structures on spheres of high dimension. Can such ideas be exploited in dimension 4 to attack the smooth Poincaré conjecture?

In one dimension higher, one of the driving questions in 4-dimensional topology is the smooth Poincaré conjecture and its symplectic analog. It is rather unbelievable that topologists still don't know how many smooth structures there are on the 4-sphere or the complex projective 2-space, and which admit symplectic structures. There has recently been a burst of activity in this area. Michael Freedman, Robert Gompf, Scott Morrison, and Kevin Walker have shown how to use Khovanov homology to get an obstruction to specific handle decompositions of homotopy 4-spheres being the actual 4-sphere (that is this obstruction could identify a counterexample to the smooth Poincaré conjecture, if it exists!). After this work Selman Akbulut and Robert Gompf showed that many potential counterexamples to the Poincaré conjecture are actually the standard sphere. Another approach to such problems is to try to build exotic smooth structures on "smaller and smaller" 4-manifolds. After Freedman and Donaldson's work in the early 1980's the problem for $CP^2 \#_n \overline{CP^2}$ could be handled for $n = 9$, After Kotschick, who handled the case $n = 8$, there was little progress made until J. Park's breakthrough a few years ago. There has since been a flurry of activity on existence of exotic smooth structures on small symplectic 4-manifolds by different teams of researchers (Akhmedov-Park, Baldridge-Kirk, and Fintushel-Stern-Park). The advances are made by exploiting a certain tension between constructions and invariants. Using clever new cut-and-paste constructions such as knot and rim surgery, together with an intimate understanding of their effect on invariants such as the Seiberg-Witten invariants, one can often deduce the presence of several (generally infinitely many) exotic smooth structures. The constructions ideally involve modifying the 4-manifold so as to alter the invariants without destroying the symplectic structure or homeomorphism type. This requires one to perform surgeries along particularly well-chosen surfaces embedded in the 4-manifold. It is reasonable to expect further progress on this important problem for some additional small symplectic 4-manifolds (e.g. $\mathbb{CP}^2$, $\mathbb{CP}^2 \# \overline{\mathbb{CP}}^2$, or $S^2 \times S^2$) via the various approaches that have been developed and the continued influence of the powerful 4-manifold invariants arising from gauge theory and symplectic geometry.

Contact structures on 3-manifolds and Heegaard-Floer theory: The existence of tight contact structures on 3-manifolds has been an important subject of investigation for a long time and, since the year 2000, significant progress has been made in our understanding of which 3-manifolds admit tight contact structures. This fundamental question has potential applications not only to contact geometry but also low-dimensional topology and dynamics. It also illustrates very well the natural interactions between the invariants described above and constructive methods. After many incremental steps by several mathematicians, Lisca and Stipsicz have completely classified which Seifert fibered 3-manifolds admit a tight contact structure. Their approach relies heavily on Heegaard-Floer homology through a non-vanishing criterion for the contact invariant of Ozsváth and Szabó for Seifert fibred manifolds. On the other hand, geometric methods reminiscent of the theory of normal surfaces of Haken and Kneser have led Colin, Giroux and Honda to general results such as: (1) Every 3-manifold has only finitely many homotopy classes of 2-plane fields which carry tight contact structures. (2) Every closed atoroidal 3-manifold carries finitely many isotopy classes of tight contact structures. One of the outstanding and fundamental questions here is the understanding of tight contact structures on hyperbolic 3-manifolds. Work of Kazez, Honda and Matić has led to a characterization of tight 3-manifolds in terms of right-veering diffeomorphisms, a step which should make calculations in contact homology and Heegaard Floer homology manageable, but thus far the condition of a manifold being hyperbolic has not been properly understood in this context. It is hoped that the current wide-ranging technology will help elucidate the problem of tight structures on 3-manifolds.

3 Highlights from the Workshop

A variety of geometric approaches to low-dimensional topology were represented, and several high-profile recent results in the field were featured prominently in the workshop. Here are a few key recent developments that were presented at the workshop: relations between Seiberg-Witten Floer homology and Heegaard-Floer homology, relations between various invariants of knots and of Legendrian/transverse knots, combinatorial approaches to computing Heegaard-Floer invariants and applications to various homology theories to low-dimensional topology and specifically knot/link theory.

Recently, two groups of researchers have been pursuing two different approaches to identifying Seiberg-Witten Floer homology and Heegaard-Floer homology. There is considerable interest in relating these theories on both theoretical and practical grounds, as each theory has its strengths in terms of computability and applicability. One group working on this problem, Cagatay Kutluhan, Yi-Jen Lee, and Clifford Taubes, was represented at the workshop by Kutluhan and Lee; Kutluhan gave a talk on their program to identify the two invariants [KLTI, KLTII, KLTIII]. The other group, Vincent Colin, Paolo Ghiggini and Ko Honda, were represented at the workshop by Ghiggini and Honda. They both gave talks outlining their approach to this correspondence [CGHI, CGHII].

Lenny Ng discussed joint work with Tobias Ekholm, John Etnyre and Michael Sullivan, about a new invariant of transverse knots that arose out of knot contact homology [EENS, N]. These new invariants seem particularly strong but very difficult to work with. Specifically they can distinguish most known pairs of transverse knots that have Legendrian approximations with small grid number. It was clear from the talk that much of the power of these invariants is still hidden away in the complicated algebras that describe knot contact homology, but many hints at how to extract information were discussed.

Jen Hom described an invariant associated to the knot Floer complex and used it to define a new smooth concordance homomorphism [H]. Applications include a formula for the tau invariant of iterated cables, better bounds (in many cases) on the 4-ball genus than tau alone, and a new infinite family of smoothly independent topologically slice knots.

Ciprian Manolescu gave a talk on a program, joint with Peter Ozsváth and Dylan Thurston, to combinatorially compute the Heegaard-Floer invariants of 3- and 4-manifolds. The 3-manifold work was discussed in the paper [MOT], but the 4-manifold work has yet to appear. The algorithm Manolescu described is based on presenting the manifolds in terms of links in S^3, and then using grid diagrams to represent the links. To compute the invariants, one uses certain positive domains on the grid, which can be encoded into ”formal complex structures”.

There has been little work involving contact structures on open 3–manifolds, with two notable exceptions being [E] and [T]. In his talk, Shea Vela-Vick discussed joint work with John Etnyre and Rumen Zarev defining an invariant of contact structures on open manifolds and showed that for a knot complement this new invariant corresponds to the minus version of Heegaard-Floer homology. This invariant along with the work in [T] opens to door to the exploration of contact structures on open 3–manifolds. In addition it provides new insight into the relation between sutured Heegaard-Floer theory and knot Heegaard-Floer theory and illustrates the important but mysterious role contact geometry seems to play in Heegaard-Floer theory.

4 Featured Talks

What follows is a list of the 21 one-hour talks featured at the workshop. The central themes were (some talks fit into more than one theme):

- **Twisted Alexander Polynomials.** *(Talks 1, 19)*
- **Floer Theory.** *(Talks 15, 21)*
- **Heegaard-Floer, Seiberg-Witten and/or Khovanov homology and applications.** *(Talks 5, 6, 8, 9, 11, 17, 18)*
- **Relations between homology theories.** *(Talks 2, 3, 4, 7)*
- **Knots, invariants, concordance.** *(Talks 1, 5, 6, 12, 13, 18, 20)*
- **Mapping class groups and contact structures.** *(Talks 10, 16)*
- **4-dimensional manifolds and invariants.** *(Talks 9, 14, 19)*

Below is a detailed list of speakers, titles, and brief descriptions of their talks.

1. **Stefan Friedl** (University of Cologne) *Twisted Alexander polynomials of hyperbolic knots*
   Given a hyperbolic knot we study the twisted Alexander polynomial as a function on the character variety and corresponding to the discrete and faithful representation. In particular we will discuss formal properties of such polynomials and their relation to fiberedness, chirality, the volume and the knot genus. This is based on joint work with Nathan Dunfield, Nicholas Jackson, Taehee Kim and Takahiro Kitayama.

2. **Paolo Ghiggini** (CNRS - Laboratoire Jean Leray) *From HF to ECH via open book decompositions I*
3. **Ko Honda** (University of Southern California) *From HF to ECH via open book decompositions II*
   This is a series of two talks aimed at showing an isomorphism between the hat-versions of Heegaard-Floer homology (*HF*) and of embedded contact homology (*ECH*). Heegaard-Floer homology, defined by Ozsváth and Szabó, is constructed from a Heegaard splitting of a three manifold and embedded contact homology, defined by Hutchings and Taubes, is constructed from a contact form.
   In our proof of *HF*=*ECH* we use open book decompositions as interpolating objects between Heegaard splittings and contact forms. The first step in the proof is to reduce the computation of both $\widehat{HF}$ and *ECH* to complexes defined from the page and the monodromy of the open book. Then we construct chain maps between these modified *HF* and *ECH* complexes by counting pseudo-holomorphic maps in suitably defined symplectic cobordisms. Finally we prove that the maps induced in homology are inverse of each other by degenerating the cobordisms and performing a relative Gromov-Witten computation. This is a joint work with Vincent Colin.
   In Part 1 we will explain how adapt the *ECH* complex to an open book decomposition.
   In Part 2 we will explain the construction of the chain maps between $\widehat{HF}$ and $\widehat{ECH}$.

4. **Eli Grigsby** (Boston College) *On Khovanov-Seidel quiver algebras and bordered Floer homology*
   I will discuss a relationship between Khovanov- and Heegaard Floer-type homology theories for braids. Specifically, I will explain how the bordered Floer homology bimodule associated to the double-branched cover of a braid is related to a similar bimodule defined by Khovanov and Seidel. This is joint work with Denis Auroux and Stephan Wehrli.

5. **Matt Hedden** (Michigan State University) *Unlink detection and the Khovanov module*
   Kronheimer and Mrowka recently showed that Khovanov homology detects the unknot. Their proof does not obviously extend to show that Khovanov homology detects unlinks of more than one component, and one could reasonably question whether it actually does (the Jones polynomial, for instance, does not detect unlinks with multiple components). In this talk, I'll discuss how to use a spectral sequence of Ozsvath and Szabo in conjunction with Kronheimer and Mrowka's result to settle the question (in the affirmative). This project is joint with Yi Ni, and had its birth at the Banff workshop two years ago.

6. **Jen Hom** (University of Pennsylvania) *Concordance and the knot Floer complex*
We will use the knot Floer complex, in particular the invariant epsilon, to define a new smooth concordance homomorphism. Applications include a formula for tau of iterated cables, better bounds (in many cases) on the 4-ball genus than tau alone, and a new infinite family of smoothly independent topologically slice knots. We will also discuss various algebraic properties of this construction, including a total ordering, a "much greater than" relation, and a filtration.

7. **Cagatay Kutluhan** (Columbia University) *Heegaard Floer meets Seiberg–Witten*
Recently Yi-Jen Lee, Clifford Taubes, and I have announced a proof of the conjectured isomorphisms between Heegaard Floer and Seiberg–Witten Floer homology groups of a 3-manifold. The purpose of this talk is to outline our construction of these isomorphisms.

8. **Tye Lidman** (UCLA) *Heegaard Floer Homology and Triple Cup Products*
We use the recent link surgery formula of Manolescu and Ozsváth as well as the theory of surgery equivalence of three-manifolds due to Cochran, Gerges, and Orr to relate Heegaard Floer homology to the cup product structure for a closed oriented three-manifold. In particular, we give a complete calculation of the infinity flavor of Heegaard Floer homology for torsion $Spin^c$ structures with mod 2 coefficients. This establishes an isomorphism with Mark's cup homology, mod 2, a homology theory defined solely using the triple cup product form.

9. **Ciprian Manolescu** (UCLA) *A step-by-step algorithm to compute 3- and 4- manifold invariants*
I will describe an algorithm for computing the Heegaard Floer invariants of three- and four-manifolds (modulo 2). The algorithm is based on presenting the manifolds in terms of links in S^3, and then using grid diagrams to represent the links. To compute the invariants, one uses certain positive domains on the grid, which can be encoded into "formal complex structures". One needs to check that all formal complex structures on the grid are homotopic - this is known to be true for certain grids called sparse, and conjectured to hold in general. The talk is based on joint work with P. Ozsvath and D. Thurston.

10. **Dan Margalit** (Georgia Institute of Technology) *Combinatorics of Torelli groups*
The Torelli group of a surface is the subgroup of the mapping class group consisting of elements that act trivially on the homology of the surface. One interesting subgroup of the Torelli group is the set of elements commuting with some hyperelliptic involution. It has been conjectured that this subgroup is generated by Dehn twists. I will present some progress on this conjecture. A key ingredient is a new proof that the Torelli group is generated by bounding pair maps. This is joint work with Tara Brendle and Allen Hatcher.

11. **Tom Mrowka** (Massachusetts Institute of Technology) *Filtrations on Singular Instanton Knot Homology*
This talk will discuss two filtrations that arise on Singular Instanton Knot Homology that refine the spectral sequence beginning with Khovanov homology and converging to the Singular Instanton Knot Homology. This is joint work with Peter Kronheimer.

12. **Lenny Ng** (Duke University) *Transverse homology and its properties*
After a brief summary of knot contact homology and some of its properties, I'll describe how a contact structure induces filtrations on the underlying complex that yield an invariant of transverse knots, transverse homology (joint with Tobias Ekholm, John Etnyre, and Michael Sullivan). I'll try to provide some perspective on the mysterious nature of this invariant, with emphasis on its general behavior and comparison to previously developed transverse invariants. If time permits, I'll discuss how transverse homology might produce a new Bennequin-type bound on self-linking number.

13. **Brendan Owens** (University of Glasgow) *Alternating links and rational balls*
For a slice knot K in the 3-sphere it is well known that the double branched cover Y_K bounds a smooth rational homology 4-ball. Paolo Lisca has shown that this condition is sufficient to determine sliceness for 2-bridge knots, and that this generalizes to 2-bridge links. I will discuss the problem of determining whether Y_L bounds a rational ball when L is an alternating link.

14. **Jongil Park** (Seoul National University) *A classification of numerical Campedelli surfaces*
In order to classify complex surfaces of general type with $p_g = 0$ and $K^2 = 2$ (such surfaces are usually called numerical Campedelli surfaces), it seems to be natural to classify them first up to their topological types. It has been known by M. Reid and G. Xiao that the algebraic fundamental group π_{alg} of a numerical Campedelli surface is a finite group of order ≤ 9. Furthermore the topological fundamental groups π_1 for any numerical Campedelli surfaces are also of order ≤ 9 in as far as they have been determined. Hence it is a natural conjecture that $|\pi_1| \leq 9$ for all numerical Campedelli surfaces. Conversely one may ask whether every group of order ≤ 9 occurs as the topological fundamental group or as the algebraic fundamental group of a numerical Campedelli surface. It has been proved that the dihedral groups D_3 of order 6 or D_4 of order 8 cannot be fundamental groups of numerical Campedelli surfaces. Furthermore, it has also been known that all other groups of order ≤ 9, except $D_3, D_4, \mathbb{Z}/4\mathbb{Z}, \mathbb{Z}/6\mathbb{Z}$, occur as the topological fundamental groups of numerical Campedelli surfaces. Unlike the case of topological fundamental group, there is also a known numerical Campedelli surface with $H_1 = \mathbb{Z}/6\mathbb{Z}$ (in fact $\pi_{alg} = \mathbb{Z}/6\mathbb{Z}$). Therefore all abelian groups of order ≤ 9 except $\mathbb{Z}/4\mathbb{Z}$ occur as the first homology groups (and algebraic fundamental groups) of numerical Campedelli surfaces. Nevertheless, the question on the existence of numerical Campedelli surfaces with a given topological type was completely open for $\mathbb{Z}/4\mathbb{Z}$. Recently Heesang Park, Dongsoo Shin and myself constructed a new minimal complex surface of general type with $p_g = 0, K^2 = 2$ and $H_1 = \mathbb{Z}/4\mathbb{Z}$ (in fact $\pi_{alg} = \mathbb{Z}/4\mathbb{Z}$) using a rational blow-down surgery and a Q-Gorenstein smoothing theory, so that the existence question for numerical Campedelli surfaces with all possible algebraic fundamental groups are settled down. In this talk I'd like to review how to construct such a numerical Campedelli surface.

15. **Tim Perutz** (University of Texas-Austin) *The Fukaya category of the punctured 2-torus*
In effect, Heegaard Floer theory takes place invokes the Fukaya category of the g-fold symmetric product of a genus g surface, with a filtration arising from a basepoint. The structure of this category is non-trivial to describe even in the genus-one case, and that is the subject of this talk. The filtered Fukaya category of the torus is generated by two circles, but it carries an interesting A-infinity structure. We use Hochschild cohomology to show that A-infinity structures on the relevant algebra are classified by two parameters in the ground ring. An Ext-algebra of two sheaves on a Weierstrass cubic curve carries an A-infinity structure of the right sort, and the coefficients g_2 and g_3 of the curve can be identified with our two parameters. In this way, the Fukaya category of the punctured torus (the "HF-hat" category) embeds into the dg category of perfect complexes on some cubic curve - in fact, a nodal cubic. Is this a hint of a theory mirror to Heegaard Floer cohomology? This is joint work with Yanki Lekili.

16. **Olga Plamenevskaya** (State University of New York at Stony Brook) *Planar open books, monodromy factorization and symplectic fillings*
A theorem of Wendl says that if a contact structure admits a planar open book (S, ϕ), all its Stein fillings arise from factorizations of the *given* monodromy ϕ as a product of positive Dehn twists. To obtain applications of this result, we develop combinatorial techniques to study positive monodromy factorizations in the planar case. As a corollary, we classify symplectic fillings for all contact structures on L(p,1), and detect non-fillability of certain contact structures on Seifert fibered spaces. (joint with J. Van Horn- Morris.)

17. **Dylan Thurston** (Barnard College, Columbia University) *Heegaard Floer homology is natural*
The easiest statement of invariance for Heegaard Floer homology gives an isomorphism class of groups for each 3-manifold. Can this be improved (like ordinary homology) to give an actual group, rather than an isomorphism class? We show that *HF* homology does associate a group to a based 3-manifold, giving, for instance, an action of the based mapping class group. In the proof, there is one new move on Heegaard diagrams that had not been previously checked.

18. **David Shea Vela-Vick** (Columbia University) *Contact geometry and Heegaard Floer invariants for noncompact 3-manifolds*
I plan to discuss a method for defining Heegaard Floer invariants for 3-manifolds. The construction is inspired by contact geometry and has several interesting immediate applications to the study of tight contact structures on noncompact 3-manifolds. In this talk, I'll focus on one basic examples and indicate how one defines a contact invariant which can be used to give an alternate proof of James

Tripp's classification of tight, minimally twisting contact structures on the open solid torus. This is joint work with John B. Etnyre and Rumen Zarev.

19. **Stefano Vidussi** (University of California Riverside) *Refined adjunction inequalities for 4-manifolds with a circle action*
Given a smooth 4-manifold M, there is an estimate on the minimal genus among representatives of a class of $H_2(M)$ in terms of an adjunction inequality involving Seiberg-Witten basic classes. In spite of the importance of such inequality in various problems (e.g. the solution of Thom Conjecture) it is known that in general such inequality is not sharp. In particular, in 1998, Peter Kronheimer proved that such inequality can be sharpened for 4-manifolds of the form $S^1 \times N^3$ using the Thurston norm of N. It is not clear how to extend Kronheimer's approach to other classes of manifolds.
Here we discuss how, using an approach that is quite different from Kronheimer's, we can recast and extend such result to 4-manifolds that are circle bundles over a 3-manifold whose fundamental group satisfies certain group-theoretic properties. More specifically, this group must be virtually RFRS; for example in the case of Haken hyperbolic manifolds (with $b_1 > 1$) this is a consequence of Dani Wise's program. The talk is based on joint work with Stefan Friedl.

20. **Liam Watson** (UCLA) *Decayed knots and L-spaces*
This talk introduces the notion of a decayed knot, a property derived from the left-orderability of the fundamental group of the knot. Decayed knots (1) have sufficiently positive surgeries with non-left-orderable fundamental group and (2) admit decayed cables, for sufficiently positive cabling parameters. This behaviour closely mirrors the behaviour of L-space surgeries on knots in the three-sphere. Indeed, known examples of decayed knots are L-spaces knots. This is joint work with Adam Clay.

21. **Katrin Wehrheim** (Massachusetts Institute of Technology) *Quilted Floer homology - transversality and applications*
I can briefly state a new, improved, and actually proven transversality for quilted Floer homology. From there, I can explain two recent applications: a) SU(n) invariants for 3-manifolds with a homotopy class of maps to S^1; which use a version of Cerf theory for Morse functions to S^1 with connected fibers. b) calculation of Floer homology for the Chekanov-Polterovich torus in $S^2 \times S^2$; which uses strip shrinking for immersed geometric composition and a weak removal of singularity for figure eight bubbles.

5 Scientific Progress Made

The workshop brought together leading experts from several different areas, and this sparked much scientific interaction. There were many very interesting talks proposed, and in making up the final schedule, the organizers tried to allow sufficient time for informal scientific discussions in order to facilitate interactions between the subject areas. This was accomplished by scheduling enough break time throughout the talk timetable and some longer breaks during the day to encourage as much informal open-ended discussions as possible. The evenings provided collaborating teams of researchers time to meet and discuss their research projects.

There were a number of new results that were proved at the workshop or whose proof was stimulated by conversations held during the workshop. Some of these came out of long-term collaborative projects, others from newly formed collaborations, and some came from ideas stimulated by talks and other interactions at the workshop.

Recently, using the language of Heegaard Floer knot homology two invariants were defined for Legendrian knots. One — the so called *grid invariant* — in the standard contact 3-sphere defined by Ozsvath, Szabo and Thurston [OST] in the combinatorial settings of knot Floer homology, and the other by Lisca, Ozsvath, Stipsicz and Szabo [LOSS] — known as the LOSS invariant — in knot Floer homology for a general contact 3–manifold. Both of them also give an invariant of transverse knots, in fact they were the first such invariants. The definitions of these invariants are quite different, but it has been conjectured since their initial definition that they are indeed the same. During the conference John A. Baldwin and David Shea Vela-Vick, and Vera Vértesi, completed a program showing that the above two definitions give the same invariant in the standard contact 3-sphere. The ideas formulated at BIRS further led to an alternate definition of these invariants which is more natural from the perspective of transverse knot theory. The approach is to give a new characterization of the invariants for transverse braids as the bottommost elements with respect to the filtration of knot Floer homology given by the axis. This work in still in progress, but is at a very promising stage. Discussing this work with John Etnyre, Etnyre revealed a program he and a student Bulent Tosun had to also establish the equivalence of these invariants. This approach involved generalizing the notion of grids to all 3-manifolds and seeing how both the grid and LOSS invariants fit into this picture. Prompted by discussions at the workshop Etnyre pushed this program forward and now believes it is close to fruition. The two different approaches by the two different groups promise new insight into these important new invariants and it is clear the BIRS workshop was key to the rapid progress on both programs.

Recently John Etnyre, Shea Vela-Vick and Rumen Zarev had defined a "limit homology" for knots using a sequence of sutured manifolds and maps between their sutured Heegaard-Floer homology. They also defined a new invariant of transverse knots in this new homology. They had previously conjectured the equivalence of this homology theory and HFK^{-} as well as the transverse invariant and the LOSS invariant. At BIRS, they showed that the transverse invariant does agree with the LOSS invariant under an appropriate identification of the limit groups with the minus version of knot Floer homology. This work gives a completely new perspective on not only the LOSS invariant but the minus Heegaard-Floer groups for knots as well. There are already plans to generalize this to open 3-manifolds and initiate a study of contact structures on open 3-manifolds.

Lenny Ng started a project at BIRS with Dylan Thurston, after Thurston's talk on naturality mentioned above. Applying the naturality results in Heegaard Floer theory to the grid transverse invariant in *HFK*, they believe they can strengthen the invariant. So far it appears they can distinguish some of the Birman-Menasco transverse knots using these techniques. This is quite interesting as these examples have so far resisted all previous attempts to try to distinguish them with invariants. Ng and Thurston are also exploring a transverse version of the mapping class group.

During the BIRS workshop Matthew Hedden and Olga Plamenevskaya completed the work on their paper [HP]. The environment of BIRS turned out to be incredibly productive for Hedden and Plamenevskaya, who were able to make substantial progress and significantly strengthen their results. The paper studies contact invariants associated to rational open books and uses them to verify tightness of contact structures on manifolds obtained by surgery on bindings of open books. They were granted permission to stay at BIRS for two extra days before the workshop; this extra time allowed them to prove a theorem. Moreover, another important lemma for the paper arose from conversations Plamenevskaya had with other workshop participants, specifically John Etnyre and Jeremy Van Horn-Morris. In addition, a chance conversation between Hedden and Van Horn-Morris at breakfast inspired a simple proof that the open book with trivial monodromy is characterized by the knot Floer homology of its binding. The ideas involved may prove useful for a variety of "botany" type questions.

During the workshop, Tom Mrowka and Nikolai Saveliev got to work on their index theorem project (their third collaborator on this, Danny Ruberman, was unfortunately not present), and while at BIRS, they managed to finish it up, and a preprint has posted to the arxiv shortly after the meeting [MRS].

Ciprian Manolescu and Dylan Thurston used the time at the BIRS workshop to work on a final version of their paper [MOT] that they have written with Peter Ozsváth.

Many participants also reported starting new projects, but they are at a more preliminary state than those mentioned above. For example Tye Lidman and Liam Watson began a project pertaining to the left-orderability of graph manifold integer homology spheres. This involves and was inspired by the results mentioned in Watson's talk that related left-orderability to Heegaard Floer homology. In another example Cagatay Kutluhan mentioned conversations with Jeremy Van Horn-Morris and Gordana Matic during the BIRS workshop, indicated an application of his construction, with Yi-Jen Lee and Cliff Taubes, of the isomorphism between Heegaard Floer and Seiberg-Witten Floer homologies to symplectic filling obstructions. The expectation is to be able to prove new results about such obstructions. After the workshop in Banff, Kutluhan started working on a project based on this expectation. There were numerous other such anecdotes as well as mentions of longtime collaborators finding time to further ongoing work (notably, the workshop was a valuable opportunity for collaborators on different continents, such as Stephan Friedl and Stefano Vidussi, to get together). Lastly, the workshop was an ideal opportunity for strong young researchers to talk with more established mathematicians. In one such example, Jonathan Williams specifically noted how important conversations with Katrin Wehrheim and Tim Perutz were to his research program. In another such example, Jonathan Yazinski discussed some ideas he has for constructing exotic smooth structures on various small 4-manifolds with Jongil Park, Ron Stern and Rafael Torres. In one case, Ron Stern was able to explain to explain why the construction would not lead to the desired conclusion, namely an exotic smooth structure on $CP^2\#\overline{CP}^2$. In another case, Rafael Torres and Jonathan Yazinski worked together on an approach for modifying "numerical" constructions of algebraic surfaces to produce exotic 4-manifolds.

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