Final Report on the 5-day workshop "Volume Inequalities" (March 28 – April 2, 2010)

Volume is one of the most fundamental concepts of mathematics and in particular, of geometry. Also, it plays a central role in discrete geometry, geometric measure theory as well as in asymptotic geometric analysis. Our major goal was to discuss the possibility of further progress on a number of important research problems of the above mentioned three fields by bringing together a good number of leading experts. There have been 25 lectures on attractive recent results that on the one hand, have given a focused overview of the state of the art of the matters within the given research area on the other hand, have proposed new techniques as well as conjectured new results.

In the following we give a brief overview of a selection of lectures. Out of the 25 lectures delivered at our conference 6, 9 and 10 reported on (significant) progress on a number of (fundamental) problems of geometric measure theory, asymptotic geometric analysis and discrete geometry. Here we just highlight some of the major results discussed in the lectures and refer the interested reader for more details to the complete list of lectures and abstracts on the workshop webpage. Also, we wish to mention that almost all lectures reported on some recent results that generated informal discussions among the conference participants representing all three major research areas at focus.

<u>Geometric Measure Theory:</u> *Simon Cox*'s lecture "*The minimal perimeter for N confined deformable bubbles of equal area*" reported on candidates to the least perimeter partition of various polygonal shapes into N planar connected equal-area regions for $N < 43$. Also, candidates to the least perimeter partition of the surface of the sphere into N connected equal-area regions have been listed. For small N these can be related to simple polyhedra and for $N > 13$ they consist of 12 pentagons and $N - 12$ hexagons. *Max Engelstein*'s lecture *"The Least-Perimeter Partition of the Sphere into Four Equal Areas"* proved that the least-perimeter partition of the sphere into four equal areas is the regular tetrahedral partition. *Frank Morgan*'s lecture "*The Isoperimetric Problem in Spaces with Density"* discussed recent results on the isoperimetric problem in Euclidean space with density whenever the log of the density is convex. The lecture of *John M. Sullivan* under title "*Rope length and related packing problems"* investigated the rope length problem that considers curves of thickness at least one, and asks to minimize the tube volume (or equivalently length) within a given knot type.

<u>Asymptotic Geometric Analysis:</u> *Vitali Milman* delivered a survey lecture on the role of polarity and stability in high-dimensional convex geometry. *Emanuel Milman* proved a generalization of Caffarelli's Theorem and showed its relation to the Gaussian correlation conjecture and similar correlation inequalities for non-Gaussian measures. *Peter Pivovarov* discussed super-Gaussian bounds for the volume of caps of convex isotropic bodies and their relations to the mean-widths. *Elisabeth Werner* introduced a new affine invariant of a convex body, which can be found as the relative entropy of the cone measure of the body, and showed new affine isoperimetric inequalities. *Vlad Yaskin* presented solutions of two open problems on unique determination of convex polytopes. *Artem Zvavitch* proved that if a convex body K is close to the unit ball and the intersection body of K is equal to K, then K is the unit ball. He also discussed a harmonic analysis version of this question.

<u>Discrete Geometry:</u> *Karoly Bezdek*'s lecture entitled "*Illuminating Ball-Polyhedra"* has given an extension of the well-known theorem of Schramm on illuminating convex bodies of constant width and proved the Boltyanski-Hadwiger conjecture for fat ball-polyhedra. Here ball-polyhedra are intersections of finitely many congruent balls in Euclidean space. Moreover, the ball-polyhedron is called a fat one, if it contains the centers of its generating balls. The probabilistic method of the proof is centered around estimating the volume of convex bodies of constant width in spherical d-space. *Gabor Fejes Toth*'s lecture "*Partial covering of a convex domain with translates of a centrally symmetric convex disc"* generalized some old theorems of L. Fejes Toth and C. A. Rogers as follows. Let D be a convex domain in the plain and let S be a family of n translates of a centrally symmetric convex disc C. An upper bound was proved for the area of the part of D covered by the discs of S. The bound is best possible in the sense that it is asymptotically tight when n and the area of D approach infinity so that the density of the discs relative to D is fixed. *Igors Gorbovickis*'s lecture "*Kneser-Poulsen conjecture for*

low density configurations" was centered around the Kneser-Poulsen conjecture according to which if a finite set of balls in Euclidean d-space is rearranged so that the distance between each pair of centers does not decrease, then the volume of the union does not decrease. It was proved that if before the rearrangement each ball is intersected with no more than $d + 2$ other balls, then the conjecture holds. The central problem of *Oleg R. Musin*'s lecture "*The Tammes problem for N=13"* asked for the arrangement and the maximum radius of 13 equal size non-overlapping spheres touching the unit sphere. The lecture reported on a computer-assisted solution based on the enumeration of the so-called irreducible graphs. *Rolf Schneider*'s lecture "*A Volume inequality and coverings of the sphere"* proved that among spherically convex bodies of given inradius in spherical d-space the lune has the largest possible volume. Based on this a Tarski-type result was proved including the statement that if the d-dimensional unit sphere is covered by finitely many spherically convex bodies, then the sum of the inradii of these bodies is at least π.