Noise, time delay and balance control

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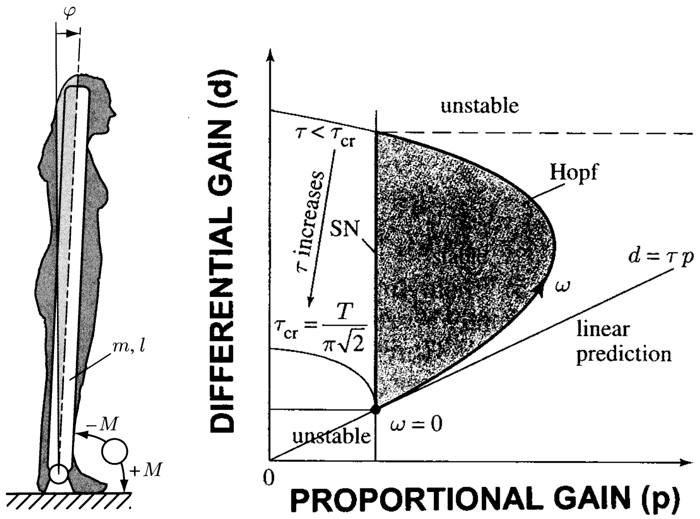
Problems related to the stabilization of an inverted pendulum are an important concern to applied mathematicians and to neuroscientists who work on balance control in humans and two–legged robots. Although feedback delay and the effect of random perturbations ("noise") pose common and challenging problems there has been little exchange in ideas between the mathematical, engineering and neuroscience communities. However, given our aging population it has become critically important that these groups work together to devise timely strategies to reduce the risk of falling in the elderly.

Traditionally the approach of neuroscientists to the study of balance control has borrowed heavily from control systems theoretic concepts developed by engineers [9, 14, 16, 23, 24, 31, 32, 33, 34]. Obviously at the most fundamental level balance is maintained by contractions of muscles that are controlled by the nervous system. These considerations have led to two types of control: 1) **closed–loop, or negative, feedback control**, in which corrective movements that are dependent on current sensory feedback, and 2) **open–loop control** in which controlling actions are much less dependent on sensory input. The necessity of open–loop control is because movements can occur faster than neural latencies (time delays), i.e. it is currently held that for fast movements the nervous system uses a **feed-forward, or predictive, type of control** to compensate for the presence of time delays: by making an internal model the nervous nervous predicts in advance when corrections are being made.

Recent studies, initially related to how the nervous system acquires expertise in the performance of a voluntary motor task, have led to the concepts of **active and passive control** (for a review see [28]). This type of control is best illustrated through the example of the thermostatic control of room temperature. Typically thermostats operate as bang–bang controllers, i.e. the furnace is either on or off. Two thresholds are chosen, one higher than the other, to reduce the number of times the furnace turns on and off. The furnace is turned on whenever room temperature falls below the lowest threshold and off when the temperature exceeds the higher threshold. Once the thresholds have been set, room temperature is self–controlled: no further in the thresholds are required to ensure that room temperature stays within the desired range. Active feedback control corresponds corresponds to the trial and error process of changing the thresholds until the appropriate operating range is determined. Since the corrective actions are only made when temperature crosses a threshold, the control mechanism is *discontinuous* and the corrective actions are applied **intermittently** [1, 2, 4, 13, 15, 25, 30]. In contrast to the control theoretic concepts imported from engineering by neuroscientists which are based on linear stability theory, the concept of discontinuous, intermittent control is an essentially nonlinear control strategy. The advantage of discontinuous types of control is that they are optimal in the presence of time delays and noisy perturbations.

This unique workshop allowed investigators to 1) directly compare mathematical predictions to experimental observations; 2) devise new experimental paradigms to test mathematical findings; and 3) discuss how



(a) (b)

Figure 1: (a) A simple one degree of freedom model of postural balance in which the ankle joint provides the necessary control torque M. (b) The shaded stability region in the parameter plane for the delay–differential equation for the torque control of balance shown in (a). For more details see [39].

workable strategies can be devised to improve balance control in humans and in two–legged robots. A motivating theme of the workshop was to compare and contrast two commonly studied experimental paradigms of human balance control: postural sway during quiet standing and stick balancing at the fingertip.

1 Mathematical Background

Postural sway during quiet standing and stick balancing at the fingertip are both stabilized by time–delayed feedback. However, the corrective forces are introduced differently. Postural sway during quiet standing represents an involuntary, or automatic, balance task in which control is effected by the direct application of torques at the pivot point (i.e. the ankle joint). In contrast, stick balancing at the fingertip is a voluntary complex motor task in which the stabilization is effected by the application of forces at the base (i.e. the fingertip). The fact that the probability that a human falls during quiet standing is much, much lower than the probability that a stick balanced at the fingertip falls suggests that there are differences in how the nervous system controls these tasks. However, despite the fact that the way in which corrective movements are made differs between these paradigms differs (compare subsection 1.1 to subsection 1.2), it is possible that similar control strategies may be at work (see subsection 1.3).

Mathematical studies of human balance control are made difficult by the fact that the precise identity of the controller is not known and hence the full dynamical system cannot be written down. Consequently the approach has been to try to use comparisons between experimental observations and propose models to guess the nature of the controller. These efforts lead to three types of modeling approaches.

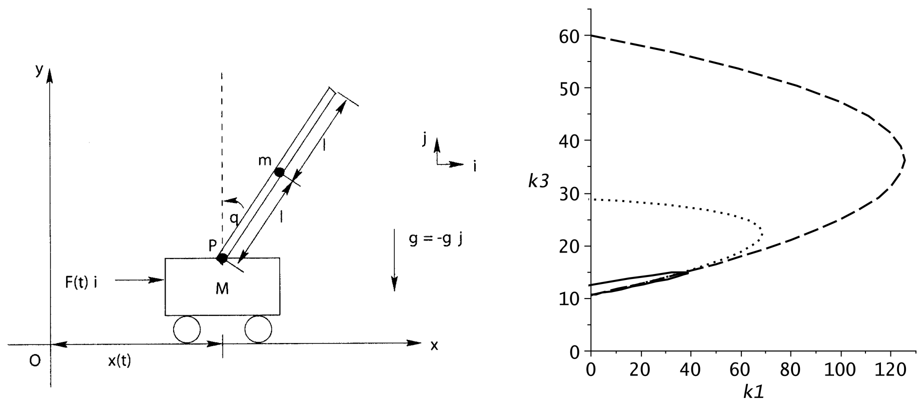
1.1 Torque control

The model for an inverted pendulum stabilized by the direct application of torque at the pivot has the general form (Figure 1a)

$$\frac{4}{3}m\ell^2\ddot{\theta} - mg\ell\sin\theta = F(\theta(t-\tau), \dot{\theta}(t-\tau)) \tag{1}$$

where m is the mass of the pendulum, ℓ is half the length of the pendulum, τ is the feedback delay and θ is the vertical displacement angle. Typically F is to be linear, i.e.,

$$F = p\theta(t-\tau) + d\dot{\theta}(t-\tau).$$



(a) (b)

Figure 2: (a) Schematic representation of inverted pendulum stabilized by the movements of a cart. M is the mass of the cart, ℓ is half the length of the pendulum, and P is the pivot point of the pendulum. See text for definition of other parameters. (b) Stability of the upright fixed point for the delay differential equation that describes (a). For more details see [17].

If the control is passive, then the torque F comes from the intrinsic mechanical impedance of the ankle joint and there is no time delay $\tau = 0$. For active control the torque F is due to feedback control exerted by the nervous system and $\tau > 0$ represents the neural transmission delay.

It has been shown that for appropriate choices of the control gains, p, d, and for τ sufficiently small, the upright position of the pendulum is stable (for a review see [39]). Figure 1b shows the possible choices as a region in the p, d parameter space. For passive control this region is given by $p > mg\ell$, $d > 0$. (Note that $p = mg\ell$ is the vertical line in Figure 1b). For active control with $0 < \tau < \tau_{cr}$ the region is the shaded region in Figure 1b. For active control with $\tau > \tau_{cr}$ there is no choice of p, d that stabilizes the upright position. Some authors [1] have suggested the feedback is a combination of active and passive control, i.e.

$$F = p_0\theta(t) + d_0\dot{\theta}(t) + p\theta(t-\tau) + d\dot{\theta}(t-\tau).$$

In this case the region of Figure 1b just gets expanded. E.g. the vertical line will be given by $p = mg\ell - p_0$.

Experimental observations support that the relevant parameters from the human neural control of balance correspond to stable regime [9, 14, 16, 23, 24, 31, 32, 33, 34]. However, this view has been challenged [1, 2, 22, 19, 20, 21].

Further, it has recently been suggested [18, 44, 45] that the active feedback should depend on the acceleration as well as the position and velocity, i.e.

$$F = p\theta(t-\tau) + d\dot{\theta}(t-\tau) + a\ddot{\theta}(t-\tau). \tag{2}$$

In this case, (1) has the form of a neutral functional differential equation, since the delay appears in the highest derivative of the state. The properties of neutral functional differential equations are not well understood; even the determination of their properties using computer simulations can be problematic especially in the presence of noisy perturbations.

1.2 Base control

The standard engineering approach to the problem of stabilizing an inverted pendulum is depicted schematically in Figure 2a. The pendulum is attached to a cart by means of a pivot, which allows the pendulum to rotate freely in the xy plane. Neglecting any friction in the pivot, the equations of motion for the full system are:

$$\begin{aligned} (m+M)\ddot{x} + F_{\text{fric}} + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}^2\sin\theta &= F(\theta(t-\tau)) \\ m\ell\ddot{x}\cos\theta + \tfrac{4}{3}m\ell^2\ddot{\theta} - mg\ell\sin\theta &= 0. \end{aligned} \tag{3}$$

where M is the mass of the cart, ℓ is the half the length of the pendulum, i.e., the distance from the pivot to the center of mass of the pendulum, and F represents the force that is applied to the cart in the x direction for the purpose of keeping the pendulum upright, i.e.

$$F_{\mathrm{control}} = k_1 x(t-\tau) + k_2\theta(t-\tau) + k_3\dot{x}(t-\tau) + k_4\dot{\theta}(t-\tau), \tag{4}$$

where it is assumed that the measurements all occur at the same time. The characteristic equation obtained by linearizing about the equilibrium point corresponding to the upright position is

$$\begin{aligned}\Delta(\lambda) \quad = \quad & \ell(m+4M)\lambda^4 + 4\ell\delta\lambda^3 - 3(m+M)g\lambda^2 - 3\delta g\lambda + \\ & e^{-\lambda\tau}((3k_4 - 4\ell k_3)\lambda^3 + (3k_2 - 4\ell k_1)\lambda^2 + 3k_3 g\lambda + 3k_1 g).\end{aligned} \tag{5}$$

where λ is the eigenvalue. Again there are ranges of the values of the k_i for which the upright position is stable [7, 8, 39]; experiment observations using a mechanical time–delayed inverted pendulum confirm these observations [17].

However, it is important to note that these results apply only if the initial values of θ are chosen sufficiently close to the upright position ($\theta = 0$). If a perturbation pushes the inverted pendulum sufficiently far from the upright position stability of the upright position is not guaranteed: either the pendulum falls or new behaviors arise. Experimental observations of a time–delayed mechanical inverted pendulum suggest that for some parameter ranges the stable equilibrium co–exists with a large amplitude oscillatory solution [41, 26].

1.3 Discontinuous control

Up to this point, modeling efforts have not taken into account the effects of ever present random perturbations ("noise"). Control–theoretic arguments suggest that intermittent, or discontinuous, feedback control is the optimal and most robust approach for stabilizing an unstable dynamical system in the presence of noise and delay [1, 2, 4, 13, 15, 25, 30]. One way to overcome these problems is to use a switch–like, or discontinuous, feedback controller which is activated only when dynamical variables cross pre–set variables. Such mechanisms have been referred to as "act and wait", "drift and act", and discontinuous controllers. An example of a very simple drift and act controller is [12, 29]

$$\dot{\theta} - \gamma\theta + \sigma^2\xi(t) = f(\theta(t-\tau)) \tag{6}$$

where the additive Gaussian white noise term $\xi(t)$ satisfies

$$\begin{aligned}\langle\xi(t)\rangle \quad &= \quad 0 \\ \langle\xi(t)\xi(t')\rangle \quad &= \quad \sigma^2\delta(t-t')\end{aligned}$$

where σ^2 is the variance and δ is the Dirac–delta function, and

$$f(\theta(t-\tau)) = \begin{cases} 0 & \text{if } |\theta| \le \Pi \\ -K & \text{otherwise} \end{cases}$$

Experimental evidence in favor of a discontinuous control mechanism for human balance includes the observation of intermittent, ballistic–type corrective movements for both postural sway [1, 2, 19, 20, 21] and stick balancing [4, 5, 10] and observations that indicate that the upright position, i.e. $\theta = 0$, is not stable in either paradigm [3, 11, 22, 26, 40, 46].

Presentation Highlights

As indicated above, three important aspects of models for balance control and stick balancing are time delays, noise and discontinuities. Thus two talks early in the workshop focussed on what is known for these types of mathematical systems. Jan Sieber (University of Portsmouth, UK) spoke on systems with time delayed switching, such as those which occur in models with delayed, discontinuous feedback. He showed that for sufficiently long delays, the dynamics near periodic orbits is generically governed by low-dimensional smooth return maps. Rachel Kuske (University of British Columbia, Canada), gave the audience an idea of how nonlinear systems with noise and time delay can give rise to complex dynamics in unexpected ways, and emphasized that one needs to consider this when trying to validate or understand a model.

A major part of the workshop were debates about three major outstanding modeling issues. The first two debates were planned in advance, based on known disputes in the modeling of postural control. The final debate (held on the last morning of the workshop) arose spontaneously out of the discussions of the previous days.

1. **Passive vs active control:** Ian Loram (Manchester Metropolitan University, UK) presented convincing data that the biomechanical properties of the musculo-skeletal system play a major role maintaining balance during quiet standing and that corrective actions are taken intermittently. Andy Ruina (Cornell University) suggested that Loram's observation might be accounted by assuming that a dead zone exists for sensory receptors, i.e. for they are unable to detect deviations from the desired upright posture unless they are sufficiently large. He also pointed out that the most successful walking robots are those that utilize simple discontinuous (intermittent) control strategies. Francisco Valero–Cuevas (USC, USA) drew attention to the fact that control problems similar to those that arise in balance arise in the context of manipulation with our fingers using a very different musculo–skeletal plant. Indeed the ability of humans to manipulate objects with the fingertips involves complex sensorimotor loops even for basic tasks, or tasks that are time critical. The general conclusion of this debate was that passive control is not enough to give stability, i.e. some active control is needed as well. In particular although during quiet standing it might be possible to utilize passive control mechanisms, once moving balance requires active control. Ramesh Balasubramaniam (McMaster University, Canada) pointed out that although it may be possible to imagine that balance control during quiet standing can be predominantly under passive control, the control of balance in all movements clearly involves active control.

2. **Predictive vs non-predictive control:** Tamás Insperger (Budapest University of Technology and Economics, Hungary) showed that delayed feedback control and predictive control are actually the same, since any predictions can only be constructed based on the available delayed state of the system. This observation turned the debate into questions of how the available output (the delayed state, for instance) should be fed back and what kind of functions should we use in the controller? Dr. Insperger argued that intermittent control was more efficient. John Milton (The Claremont Colleges, USA) argued that in the presence of noise and delay the only control mechanism that are optimal are those that utilize an intermittent, discontinuous control strategy. James Finley and Eric Perreault (Northwestern University, USA) presented experimental evidence that suggest that the relative contribution of feedback vs feedforward (predictive) control mechanisms to postural stability depends on both the level of stability provided by the environment and how the environment influences the pattern of volitional activation.

3. **Linear vs nonlinear control:** This debate help distill the fact that models with linear and nonlinear control fit different situations. Robert Peterka (Oregon Heath Sciences Center, USA), John Jeka (University of Maryland, USA), Tim Kiemel (University of Maryland, USA) and Gabor Stepan (Budapest University of Technology and Economics, Hungary) demonstrated that balance control during quiet standing was very well described by physiological models that incorporated linear differential equations with time-delayed negative feedback. Lena Ting (Emory University, USA) presented evidence that the appropriate feedback involved terms that depended on displacement, velocity and acceleration, i.e. the models take the form of a neutral functional differential equation (NFDE). During the "after hours" discussions, Sue Ann Campbell and Amy Radnuskaya were able to show that that the differential operator in Prof. Ting's NFDE model is D stable, which means that the stability is similar to that for delay differential equations. Thus if control is continuous and the system stays close to the upright position, then a linear control model is adequate to describe the behavior. However, Ramesh Balasubramaniam (McMaster University, Canada) argued that models with nonlinear control are needed to account for the statistical properties of stick balancing experiments. Moreover, John Milton (The Claremont Colleges, USA) strongly supported this line of reasoning adding that a problem not explained by current linear control theories are experimental observations that suggest that the upright fixed point is unstable. Finally, Manoj Srinivasan (Ohio State University) presented an alternative approach to deriving a model for balance control using the formalism of optimal control. Clearly, more experimental data may be needed to resolve this debate, although mathematical studies of simple models may help determine if this question can be answered by more data.

In addition, insights into several modulating factors for balance control were discussed. For example, Jason Boulet (University of Ottawa, Canada) spoke on the effects of delayed visual feedback, in the presence of noise. Minoru Shinohara (Georgia Institute of Technology, USA) on the effects of subsensory noise.

Since this workshop involved physiologists, engineers and mathematicians, an important group of presentations were tutorials, both planned and impromptu, which helped give researchers a basic understanding of the fields outside their own. Tutorials included

- stability analysis for delay differential equations (Amy Radunskaya, Pomona College, USA)
- statistical tools for studying systems with noise (John Milton The Claremont Colleges, USA)
- methods for analyzing stochastic DDEs (Toru Ohira, Sony Computer Science Laboratories, Japan)
- the basic physiology of balance control (Lena Ting, Emory University, USA)
- deadbeat linear control (Andy Ruina, Cornell University, USA)
- experimental techniques using human motor noise for system identification (Jason Kutch, University of Southern California, USA)

A quite novel part of the workshop were the demonstrations of the experimental systems given by several participants. These included both experiments for obtaining data from physiological systems:

- the thumb-finger spring compression paradigm (Francisco Valero–Cuevas, USC, USA)
- sensory illusions resulting from tendon vibration (Lena Ting, Emory University, USA)
- balance on a wobble board (Toru Ohira, Sony Computer Science Laboratories, Japan)
- functional electrical stimulation (Kei Masani, Albert H. Vette and Mark Robinson, University of Toronto, Canada)
- mechanical inverted pendulum with delayed feedback (Toru Ohira, Sony Computer Science Laboratories, Japan)

These demonstrations were especially valuable as they helped the mathematicians understand what kind of limitations are present on the amount and kind of data which is obtained from experimental systems.

Conclusions and Future Directions

Participating neuroscientists were extremely interested in understanding the mathematics behind delay differential equations. Indeed many of the neuroscientists had already developed physiologically–based mathematical models for the control of human balance and stick balancing at the fingertip in which time delays were explicitly present. Much of their interest focused on on understanding how the modeled control strategies might be destabilized, for example, by the effects of aging on the nervous system. The increasing interest of the neuroscience community in the properties and analysis of delay differential equations highlights an educational need to provide opportunities for undergraduates, graduates and researchers to have access to this material. Given that this drive is coming from experimentally oriented neuroscientists it will particularly important to develop methods to present this material in forms that are accessible to a non–mathematical audience.

Participating mathematicians were introduced to a number of puzzling experimental observations that are as yet unexplained by current models for balance control. Examples include, the speculation by neuroscientists that being on the stability boundary improves maneuverability, the increasing importance of intermittent neural control strategies, the paradoxical effects that arise from the interplay between noise and delay, and the beneficial effects of vibration and distraction on balancing skill. It also became clear that some of the issues arising from the experiments come down to rather fundamental mathematical questions that reach beyond the study of balance control. These include the following. What are the mathematical implications of a digital versus an analog neurological or physical control system? Is it possible to detect whether a system is digital or analog? Can we distinguish mathematically, based on an analysis of the time series, between fluctuations around a linearly unstable fixed point (perhaps stabilized by noise) and fluctuations about a linearly stable fixed point? Can we distinguish a continuous system from a discontinuous system when noise is present? Clearly the interaction between noise, delay and discontinuity is a field that deserves more rigorous analysis by mathematicians.

Unfortunately the global economic climate meant that two groups working on intermittent control of human postural sway could not attend the workshop, namely, Pietro Morasso (Genova, Italy) and Tasshin Nomura (Osaka University, Japan). Moreover, the discussions drew attention to the need for a future workshop(s) in about two years for experimentalists and mathematicians to meet again to compare progress that had been made addressing the various issues that had been raised at this workshop. The important take home message for mathematicians is that the study of human balance control is an exciting field in which they can expect to make major contributions to the advance of neuroscience. Thus it will be interesting to see how this story evolves.

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APPENDIX: Comments from participants

Comments from Workshop Participants

- Thanks again for the wonderful workshop last week at BIRS. It was a good collection of people and I learned much about both human balance and DDEs. Now I feel ready to write some papers about these topics!

  - Manoj Srinivasan, Ohio State University, Columbus, OH, USA

- Had several good discussions and in general am finding the conference to be quite stimulating.

  - Kreg Gruben, University of Wisconsin, Madison WI, USA

- I learned a lot AND had a good time!

  - Ami Radunskaya, Pomona College, Claremont CA, USA

- I was used to justifying my techniques to physiologists, but this tutorial gave me the opportunity to see what a more engineering oriented person would think of the techniques I am using and the assumptions I am making. This outside perspective was very valuable to me.

  - Jason Kutch, University of Southern California, Los Angeles CA, USA

- The workshop emphasized to me the importance of systematically characterizing nonlinearities in standing.

  - Tim Kiemel, University of Maryland, College Park MD, USA

- In brief, I thought the conference was excellent. The academic standard was very high and for me it was new experience to be able to listen to such high quality talks from such a good range. The informal interaction was also far superior to that normally achieved at a conference. I think the subject genuinely moved forwards in that intermittent control is now taken more seriously by the balance community than it was at the beginning of the week.

  - Ian Loram, Manchester Metropolitan University, Manchester, UK

- I thought the workshop definitely served its purpose in this respect and overall I was probably most impressed by the range of computational and experimental techniques that people use to address questions of balance control.

  - James Finley, Northwestern University, Chicago IL, USA

- I enjoyed the conference very much, and learned a lot. It gave me a lot of good idea of problems to work on.

  - Rachel Kuske, University of British Columbia, Canada