Semiparametric Adaptive Estimation in Survey Sampling

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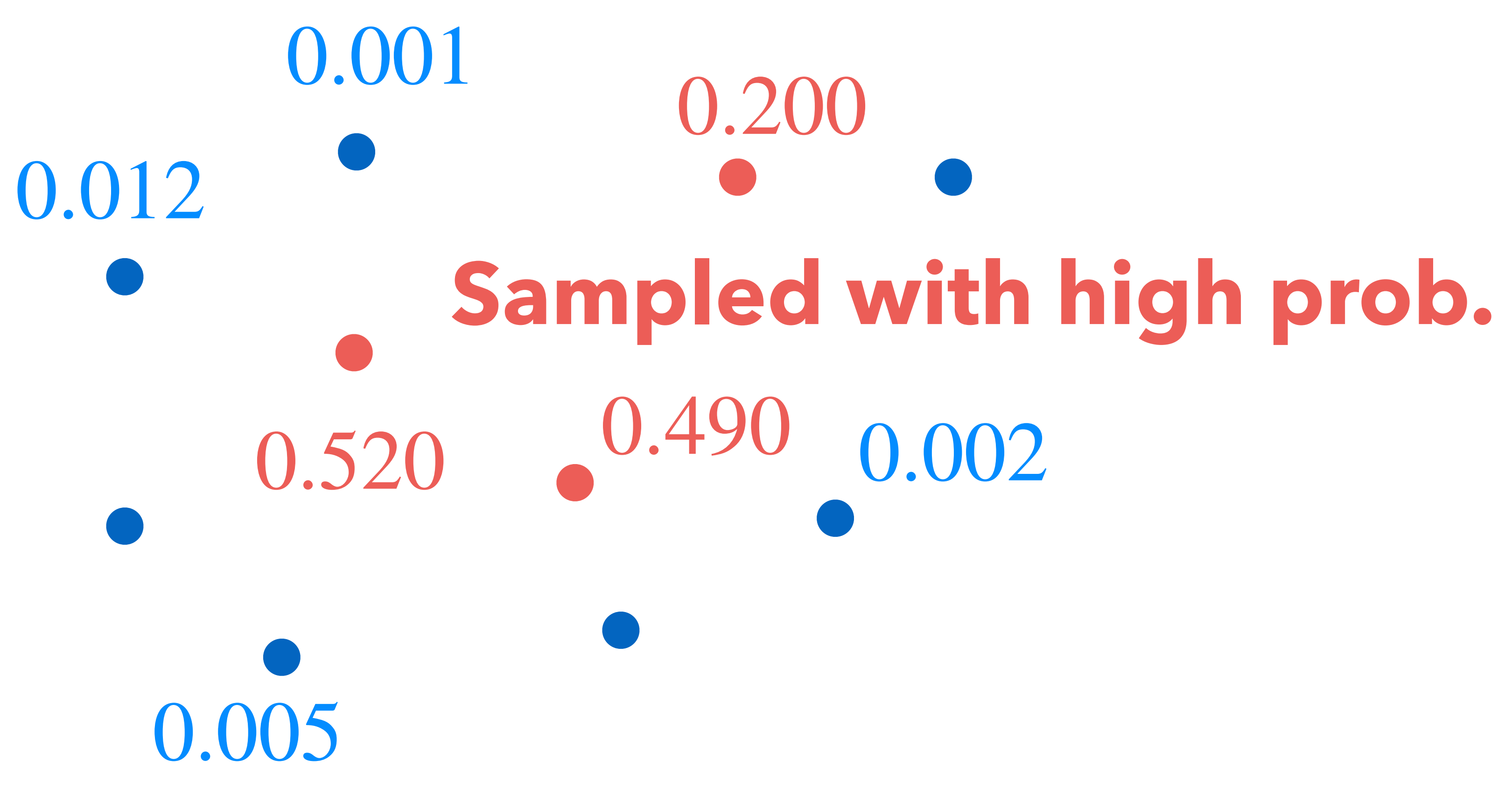
Brief Summary

- In survey sampling, some data are sampled according to inclusion probabilities instead of using all the data from the target population
- The inclusion probability (or weight) plays an important role to conduct valid statistical analysis
- However, classical weighting methods are unstable especially when the weights are extremely large
- We propose an estimator that attains the semiparametric efficiency bound by using a model on the weighting mechanism



illlustration of inclusion probability

Contents

- Introduction
- Proposed Estimator
- Simulation
- Real Data Analysis

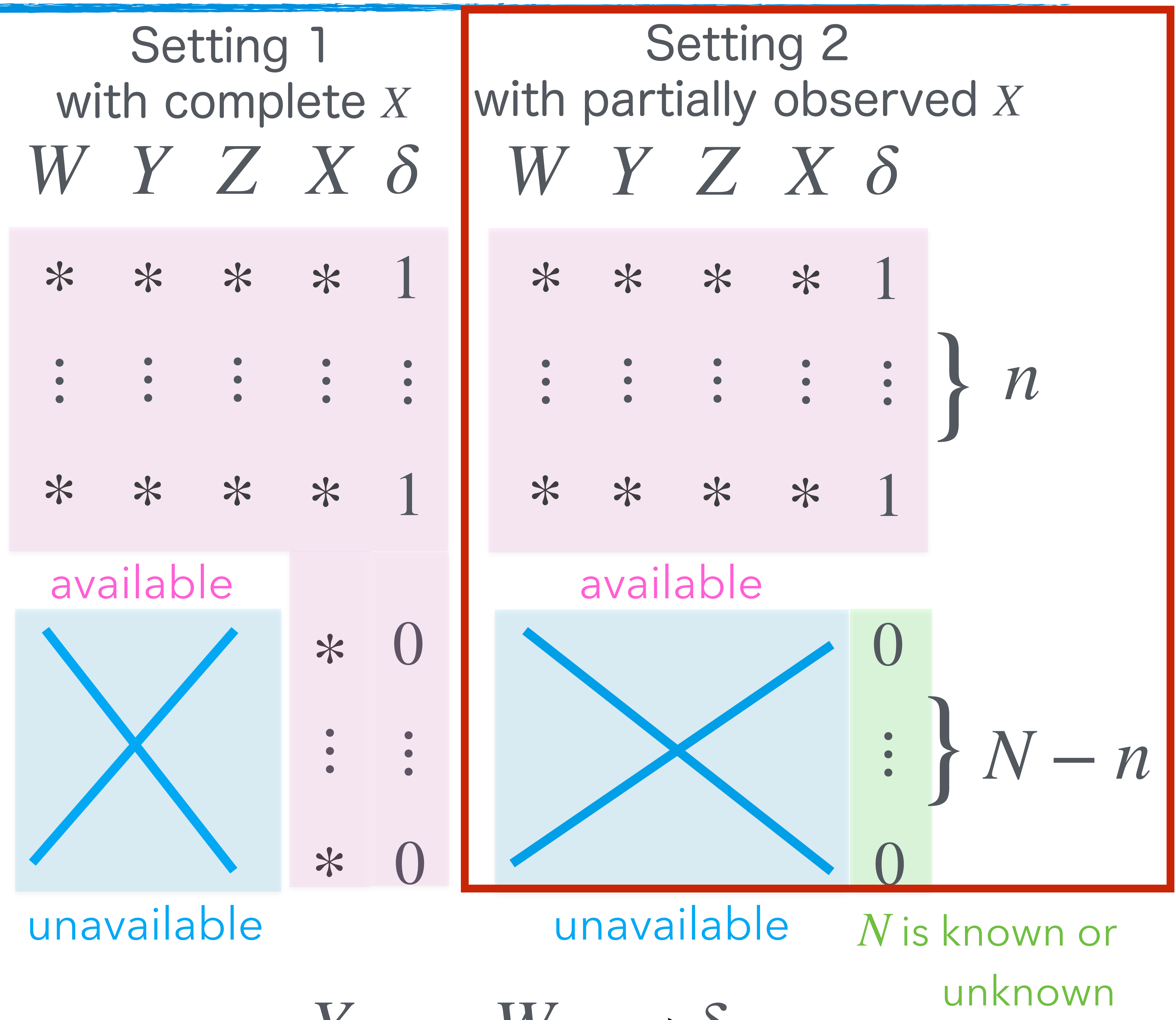
Contents

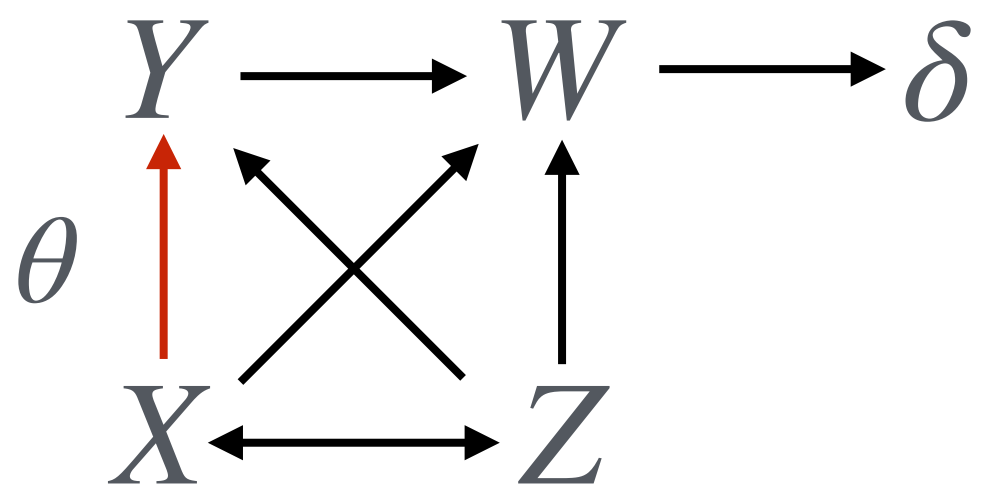
Setup

We consider this setting in this talk

- Variables: $(X_i, Y_i, Z_i, W_i, \delta_i)_{i=1}^{N} \overset{\text{i.i.d.}}{\sim} F$
  - Y: response variable
  - X: (interesting) covariate
  - Z: other covariates
  - W: inverse of inclusion probability
  - δ: sampling indicator

    takes 1 if data are sampled
  - n: size of sampled dataset $\sum_{i=1}^{N} \delta_i = n$
- Target: $E(Y)$, $E(Y \mid x; \theta)$, $f(y \mid x; \theta)$

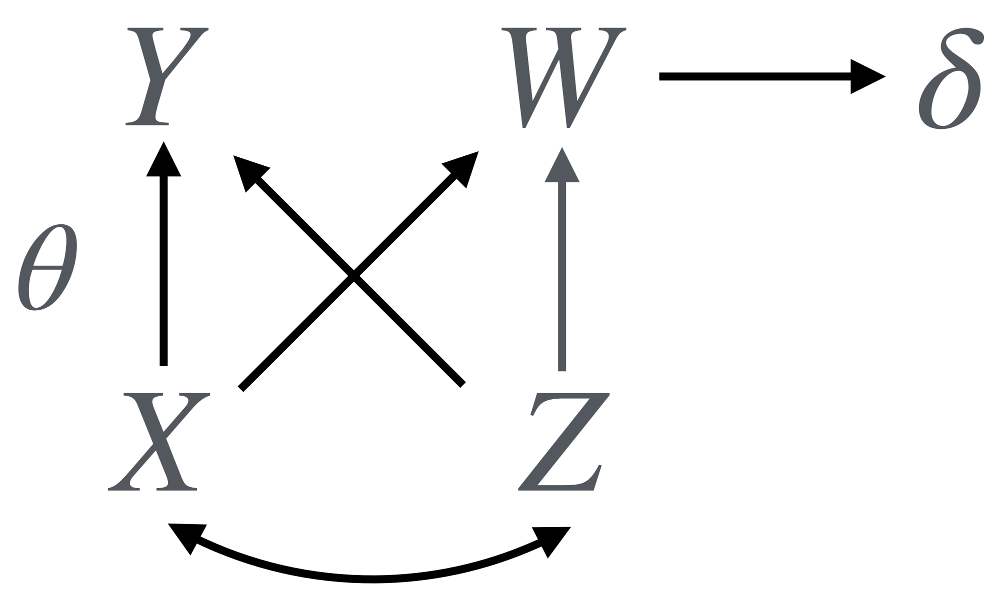




Sampling Mechanism

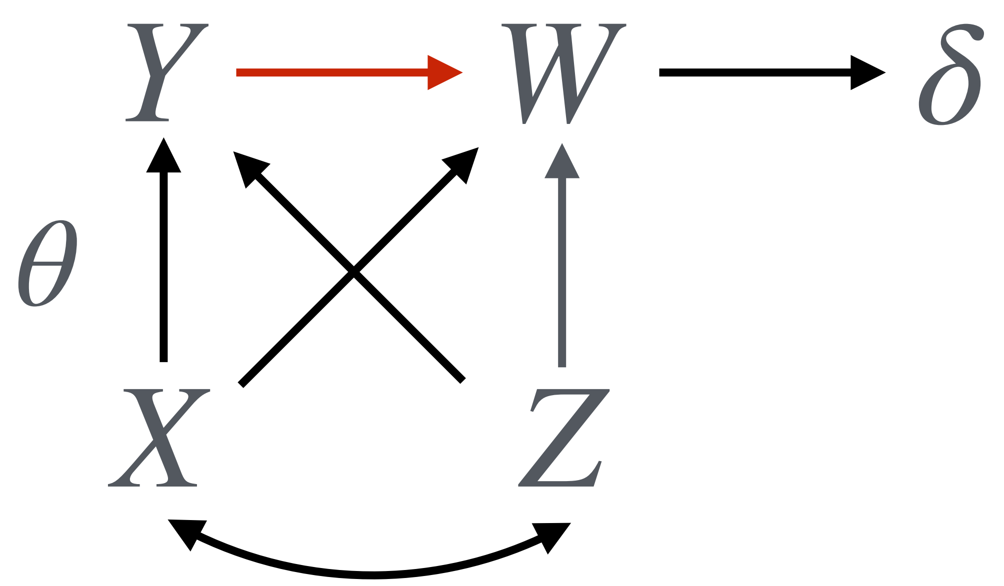
- Non-informative sampling (MAR)

$$W \perp Y \mid (X, Z)$$



- Informative sampling (NMAR)

$$W \not\perp Y \mid (X, Z)$$



We consider **informative sampling** in this talk

Example: The Canadian Workplace and Employee Survey (Fuller, 2009)

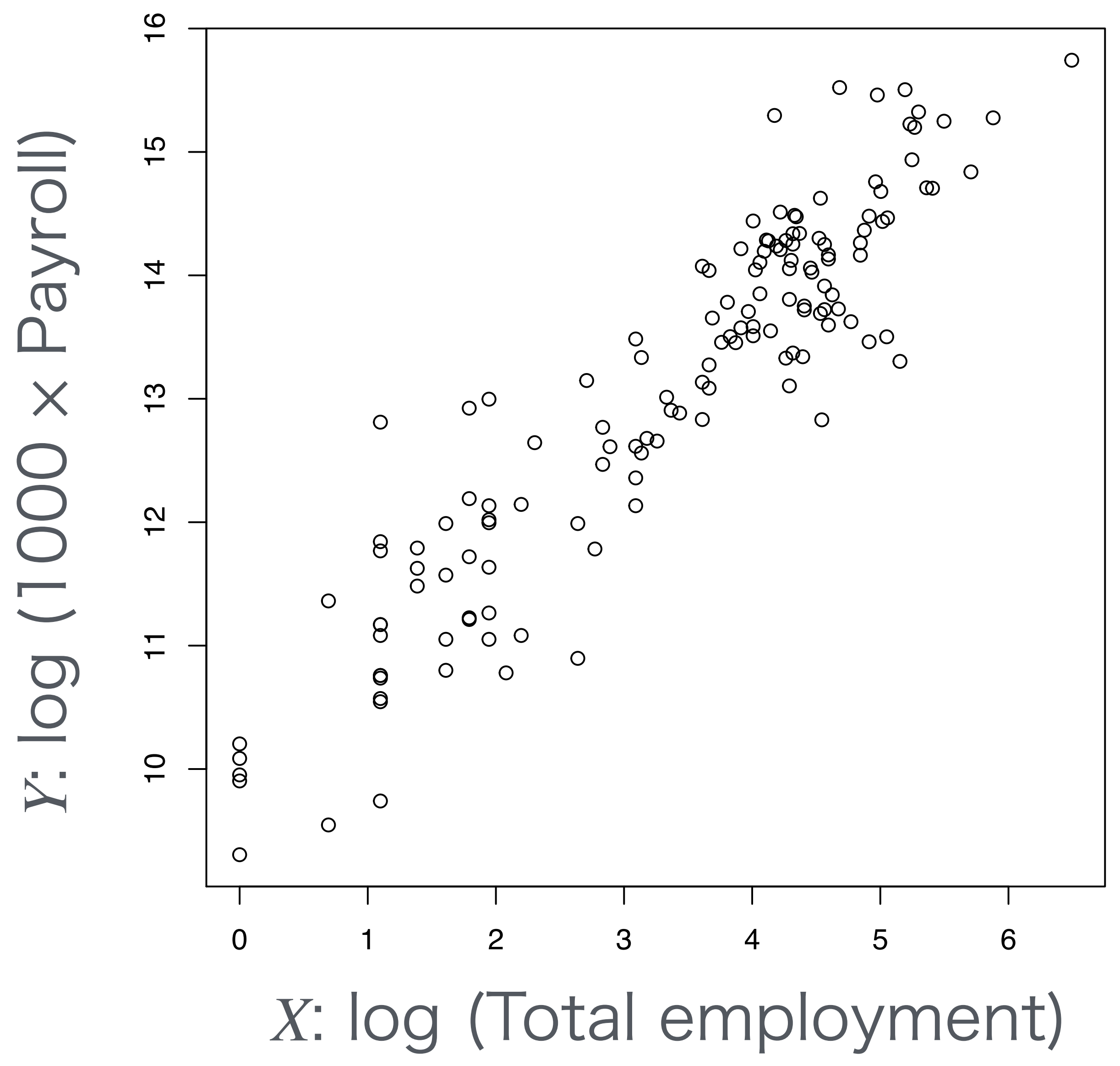
- We want to know the relationship between Payroll (Y) and total Employment (X)

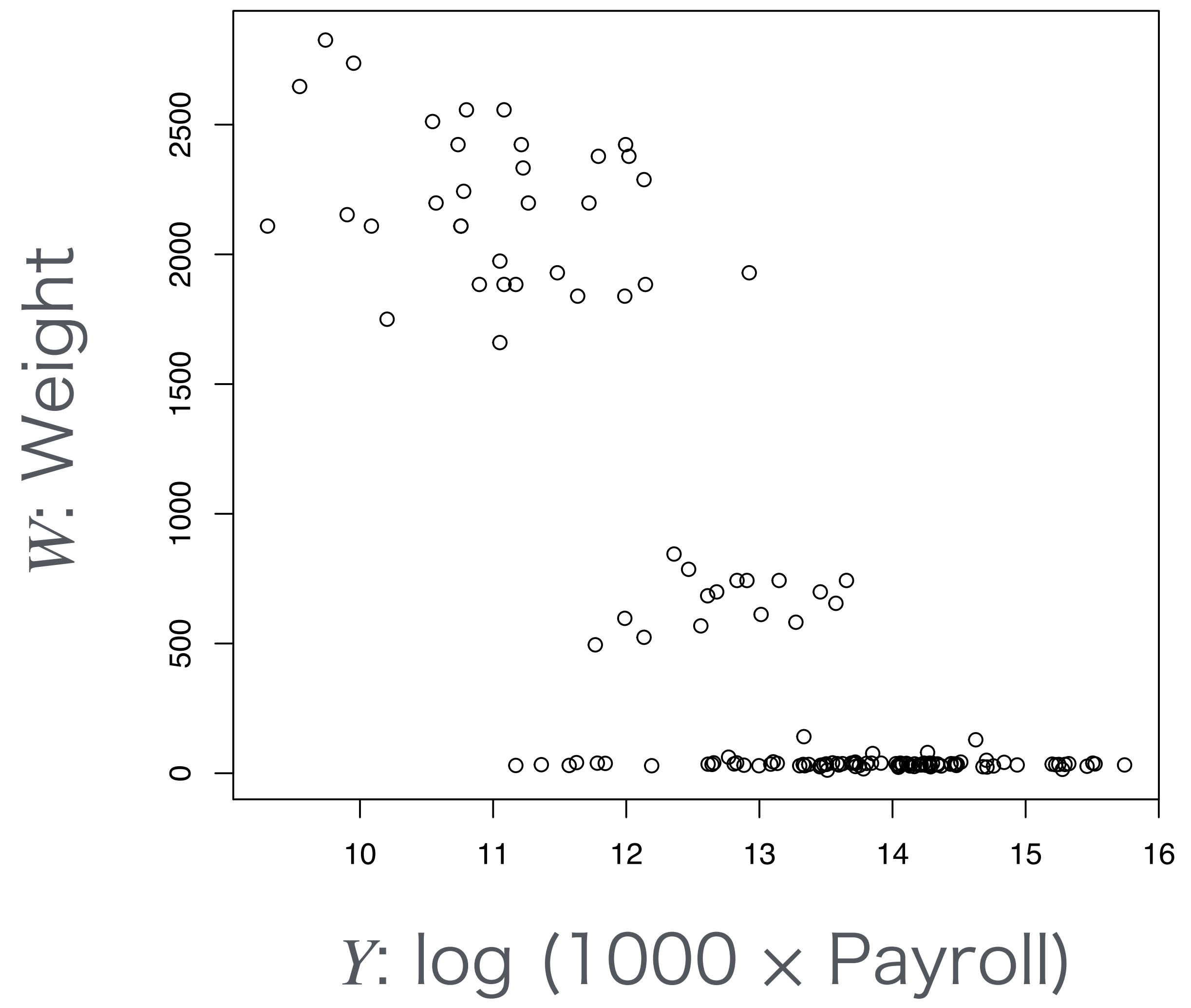
- Size of population (N): 2029 workplaces

- Sampled size (n): 142 workplaces
  - Stratified sampling (3 strata)
    + simple random sampling
    with nonresponse adjustment

- Model:

$$Y \mid X = x \sim N(a + bx,\, \sigma^2), \quad \theta = (a, b, \sigma^2)^\top$$





Z-estimator

(Semiparametric) Z-estimator θ: Unique solution to

$$E\{U(X,Y;\theta)\} = 0$$

$U(\cdot)$ depends on θ as follows..

Mean of response variable: $\theta = E(Y) \Rightarrow U(X,Y;\theta) = \theta - Y$

Regression parameter: $\mu(X;\theta) = E(Y \mid X) \Rightarrow U(X,Y;\theta) = A(X)\{Y - \mu(X;\theta)\}$

arbitrary function

Outcome model: $f(Y \mid X;\theta) \Rightarrow U(X,Y;\theta) = \frac{\partial}{\partial\theta}\log f(Y \mid X;\theta)$

$= S_\theta(X,Y)$

Score function

Horvitz-Thompson Estimator

- Horvitz-Thompson (HT) estimator: the solution to

$$\sum_{i=1}^{n} W_i U(X_i, Y_i;\ \theta) = 0,$$

Available when N is unknown

where $E\{U(X, Y; \theta)\} = 0$

- The most well known method in survey sampling
- No additional assumptions are required
- Theoretical validity: Unbiased estimating equation $\Rightarrow$ moment method

Proof for Unbiasedness

$$E\left[\sum_{i=1}^{n} W_i U(X_i, Y_i; \theta)\right] = E\left[\sum_{i=1}^{N} \delta_i W_i U(X_i, Y_i; \theta)\right]$$

$$= E\left[\sum_{i=1}^{N} \underbrace{P(\delta_i = 1 \mid X_i, Y_i, W_i)}_{= \frac{1}{W_i}} W_i U(X_i, Y_i; \theta)\right]$$

$$= N \times E\left[U(X, Y; \theta)\right]$$

$$= 0$$

Smoothing Weight

- Smoothing weight: $\tilde{W} := E(W \mid x, y, \delta = 1)$

- Beaumont (2008, Biometrika) shows that using $\tilde{W}$ instead of W is more efficient in the context of regression analysis
  - $\tilde{W}(x, y)$ is to be estimated
  - Misspecification of the model causes bias

- Kim and Skinner (2013, Biometrika) proposed an optimal weight in the same setup.

There are possibilities that we can construct more efficient estimator than HT!!

Preparation: Bayes' Theorem

- Let $f_1(y \mid x) = f(y \mid x, \delta = 1)$ and $\pi(x, y) = P(\delta = 1 \mid x, y)$

- Transformation of $f_1 \rightarrow f$

$$f_1(y \mid x) = f(y \mid x, \delta = 1) = \frac{f(y, \delta = 1 \mid x)}{P(\delta = 1 \mid x)} = \frac{f(y \mid x)\pi(x, y)}{\int f(y \mid x)\pi(x, y)\mathrm{d}y}$$

- Transformation of $f \rightarrow f_1$

$$f(y \mid x) = \frac{f_1(y \mid x)\pi^{-1}(x, y)}{\int f_1(y \mid x)\pi^{-1}(x, y)\mathrm{d}y}$$

Conditional Maximum Likelihood (CML) for Outcome model

- Assume that
  - $f(y \mid x; \theta)$ is of our interest
  - response probability $\pi(x, y) = P(\delta = 1 \mid x, y)$ is known

$$f_1(y \mid x) = f(y \mid x, \delta = 1) = \frac{f(y \mid x)\pi(x, y)}{\int f(y \mid x)\pi(x, y)\mathrm{d}y}$$

- Then, the conditional maximum likelihood (CML) estimator is the efficient: the solution to

$$\sum_{i=1}^{n} S_{1,\theta}(X_i, Y_i) := \sum_{i=1}^{n} \frac{\partial \log f_1(Y_i \mid X_i)}{\partial \theta} = 0$$

$$= \sum_{i=1}^{n} \left[S_\theta(X_i, Y_i) - \frac{\int S_\theta(x, y)\pi(x, y) f(y \mid x; \theta)\mathrm{d}y}{\int \pi(x, y) f(y \mid x; \theta)\mathrm{d}y} \right]$$

$$= \sum_{i=1}^{n} \left[S_\theta(X_i, Y_i) - E_1\{S_\theta(x, Y) \mid x; \theta\} \right]$$

How to Handle When $\pi(x, y)$ is Unknown??

- Sverchkov and Pfeffermann (1999, Sankya B) shows that

$$E_1(W \mid x, y) = \int w f_1(w \mid x, y) \mathrm{d}w \quad \frac{1}{w}$$

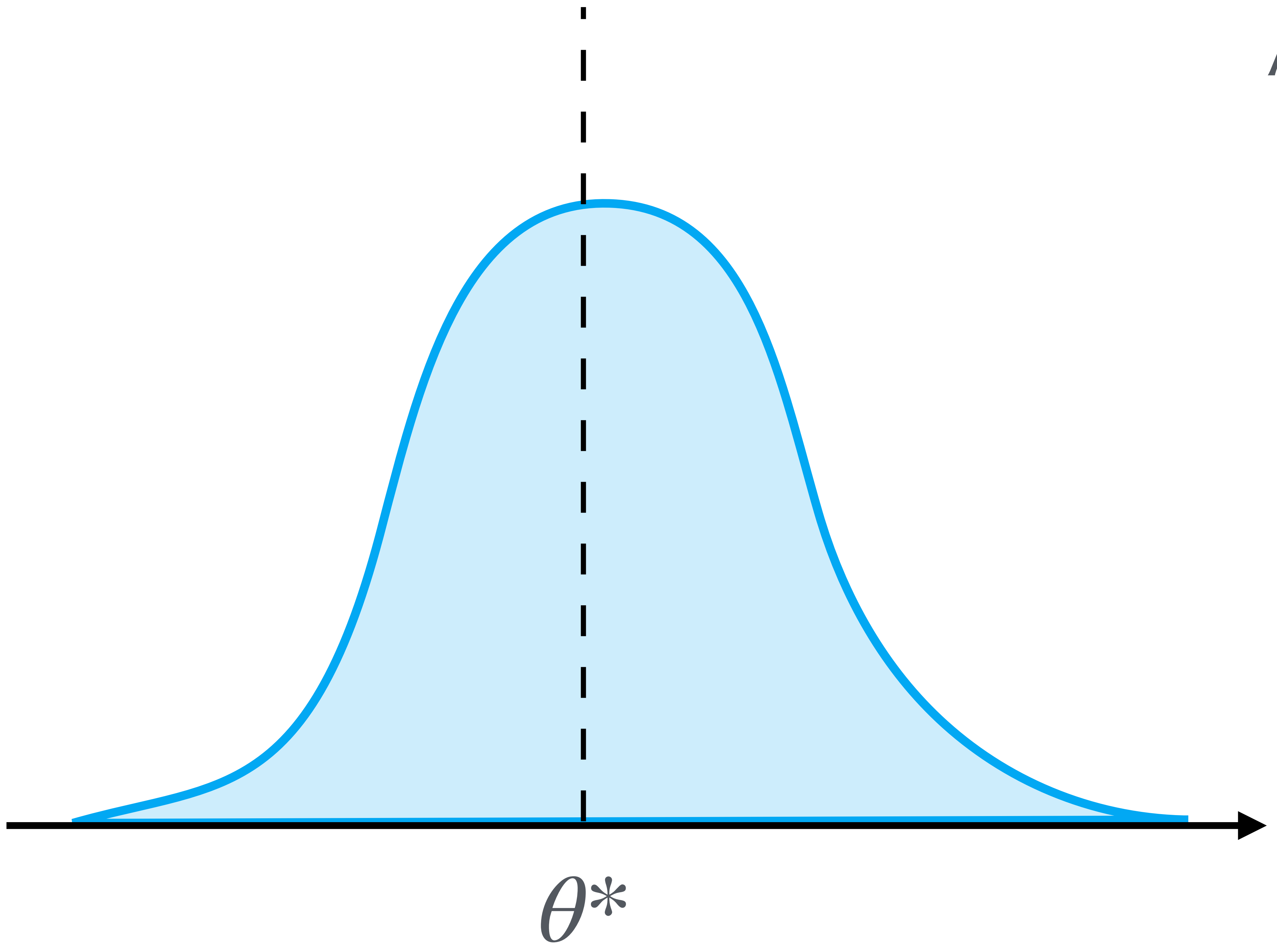
$$= \frac{\int w P(\delta = 1 \mid w, x, y) f(w \mid x, y) \mathrm{d}w}{\int P(\delta = 1 \mid w, x, y) f(w \mid x, y) \mathrm{d}w}$$

$$= \frac{1}{P(\delta = 1 \mid x, y)} =: \frac{1}{\pi(x, y)}$$

- π can be estimated by the regression W on (X, Y) with sampled data
- If π is misspecified, the estimator causes bias

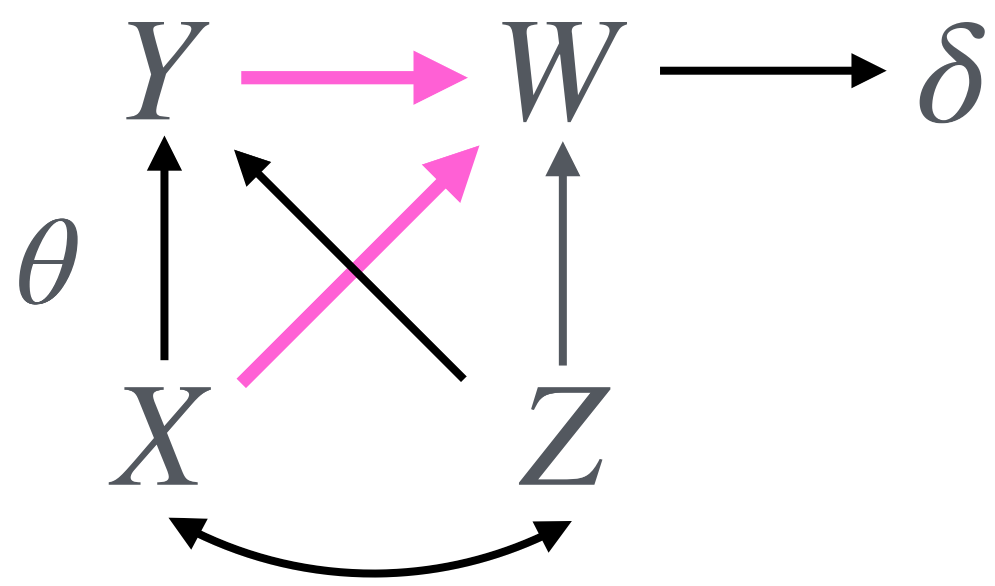
Conditional Maximum Likelihood (CML)

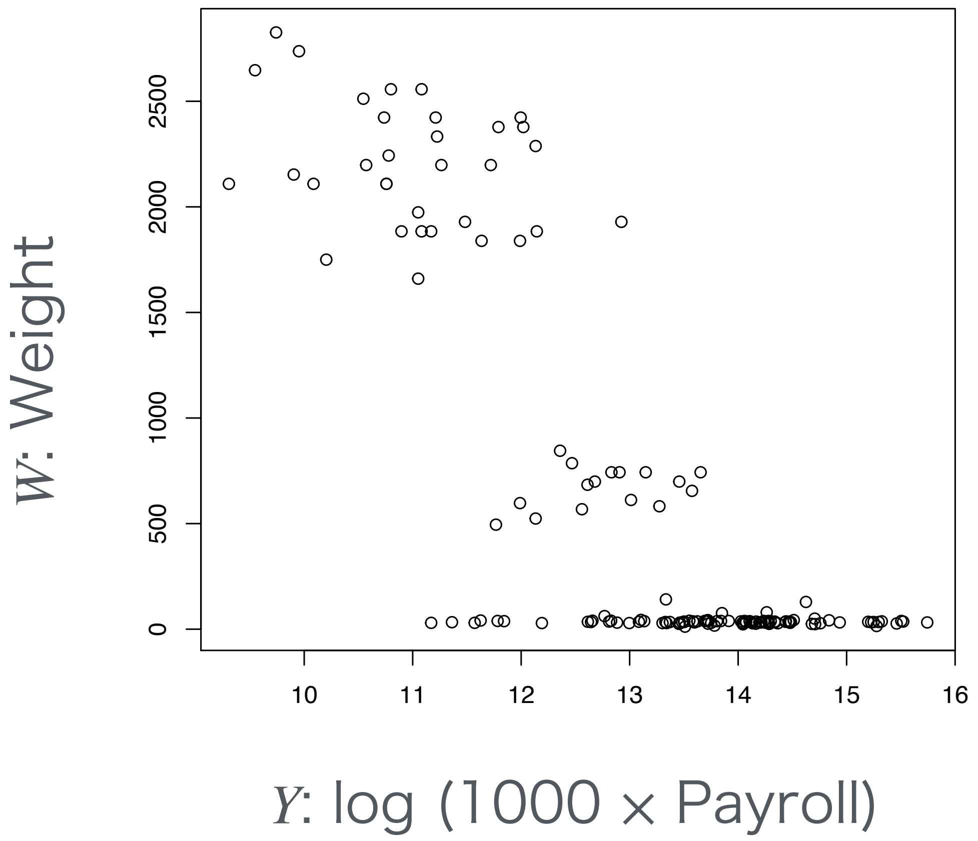
Dist. of HT estimator



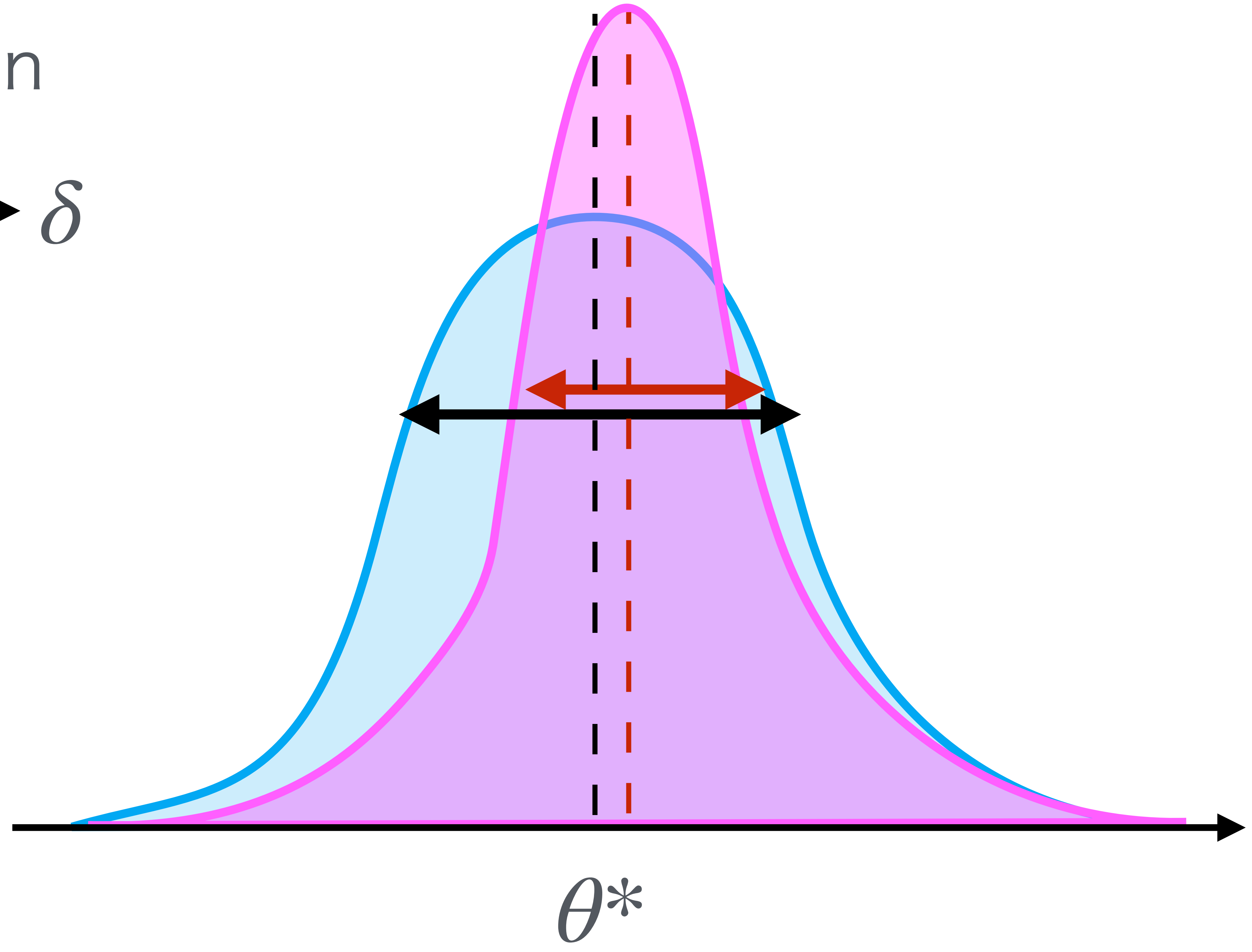
✔ Consistency
✔ Asymptotic normality

Add information on





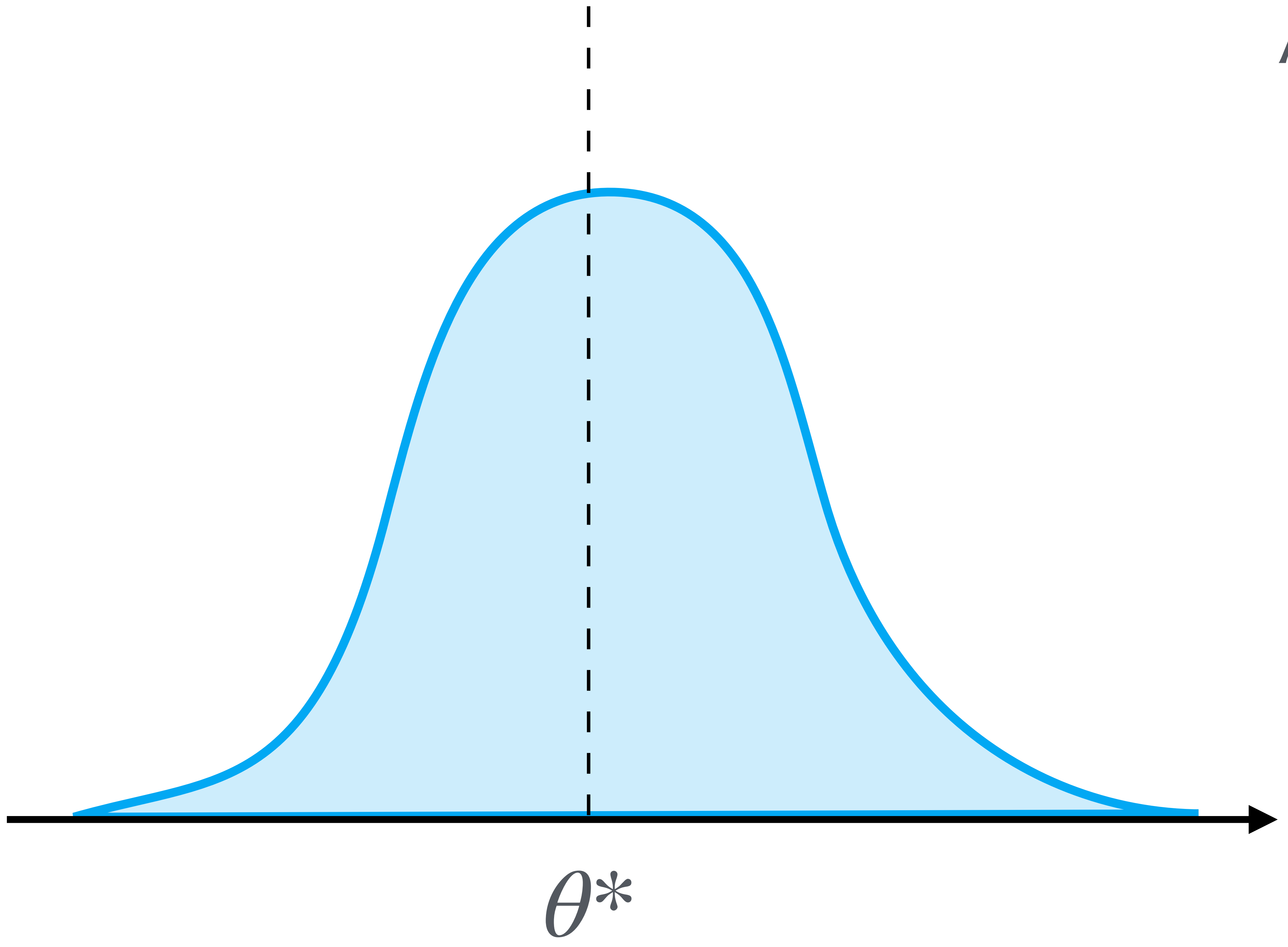
Dist. of CML estimator



✔ Consistency
✔ Asymptotic normality
✔ Efficiency

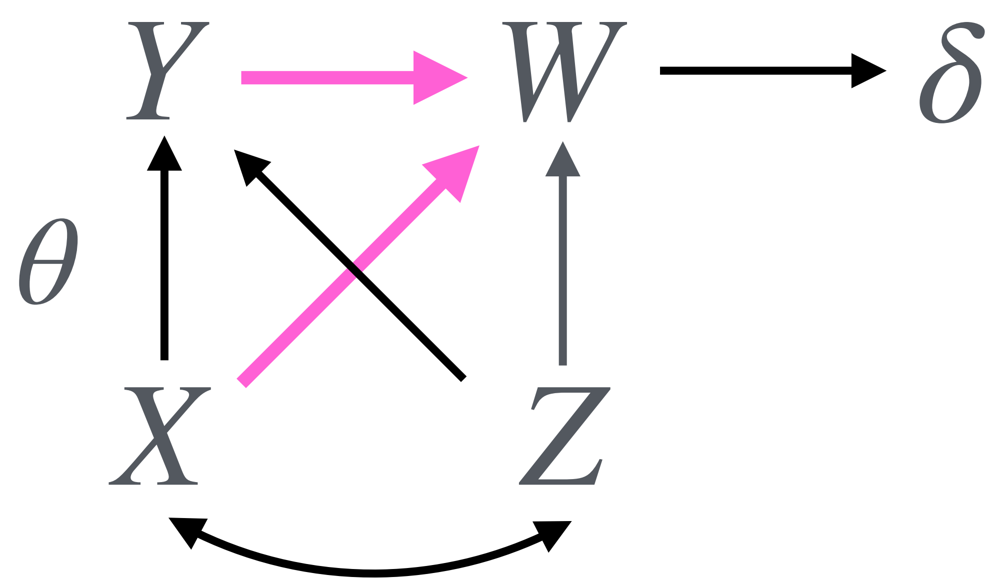
Our Goal

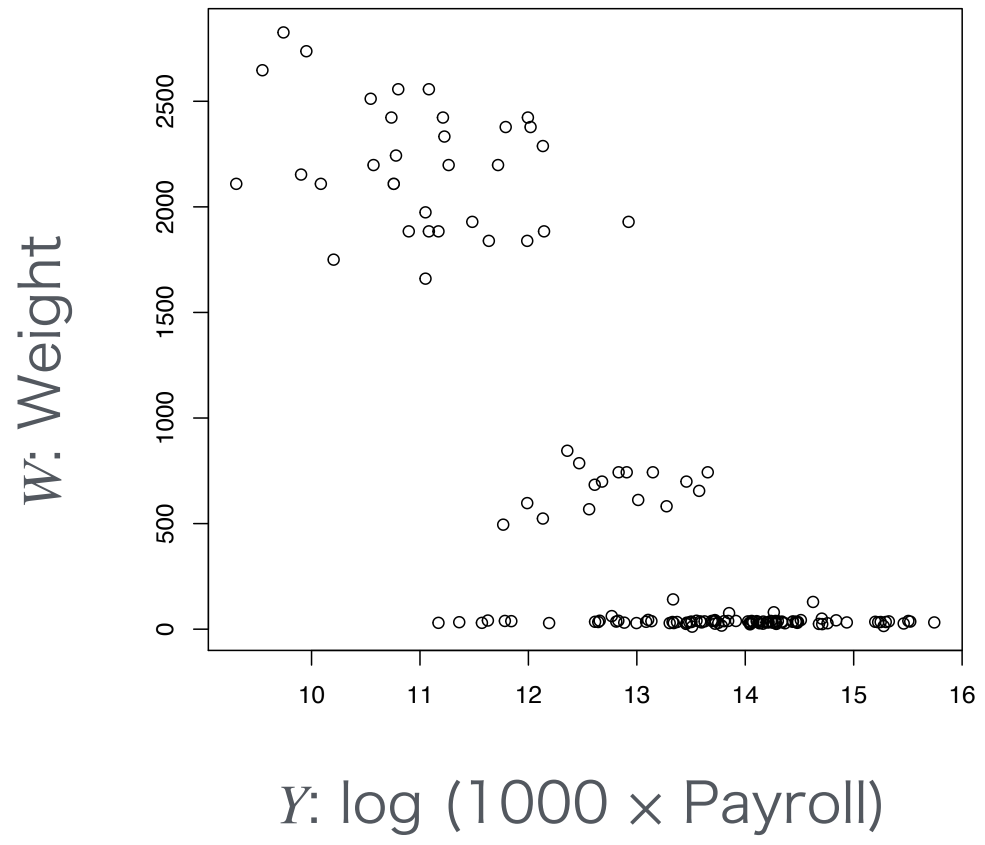
Dist. of HT estimator



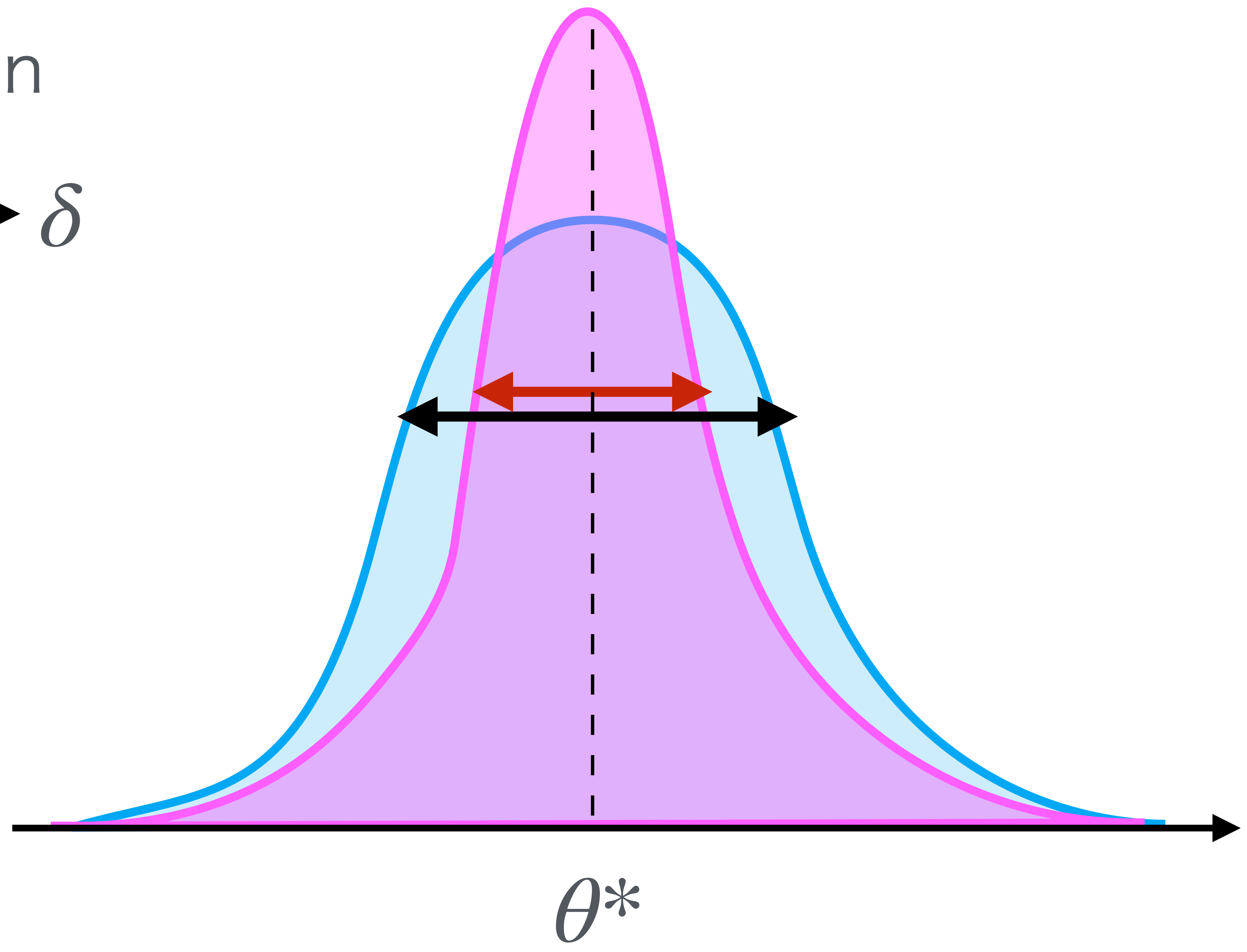
- ✔ Consistency
- ✔ Asymptotic normality

Add information on





Dist. of Proposed estimator



- ✔ Consistency
- ✔ Asymptotic normality
- ✔ Efficiency

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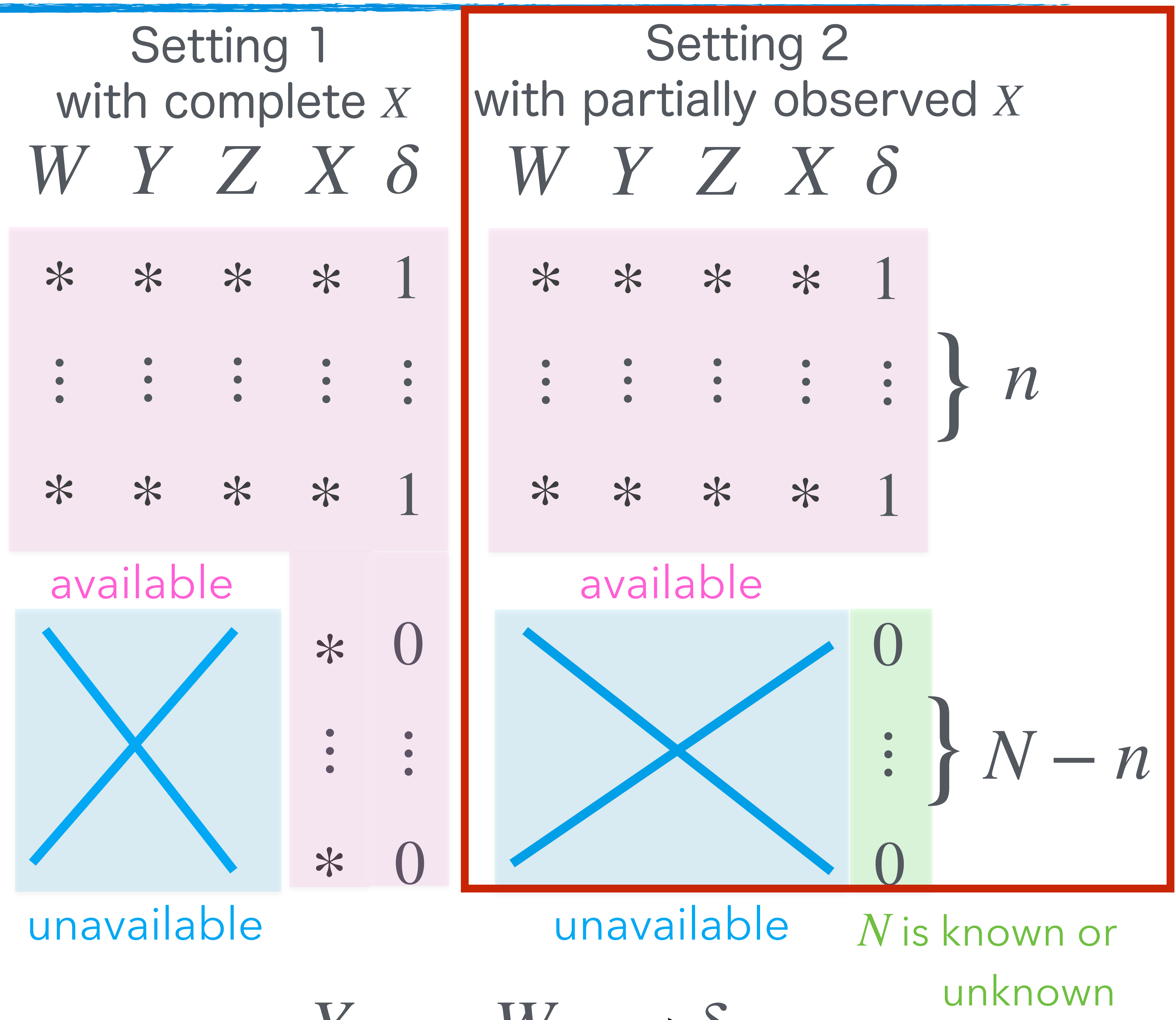
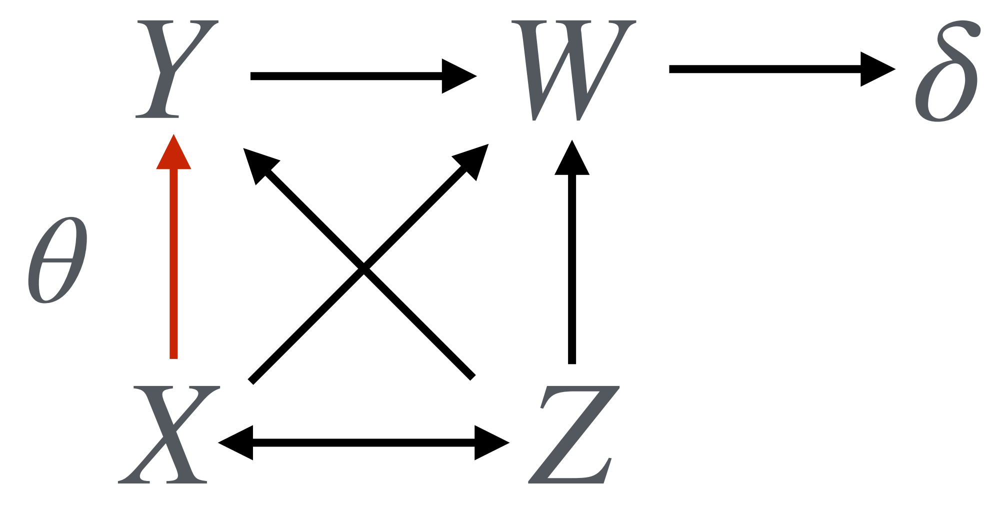


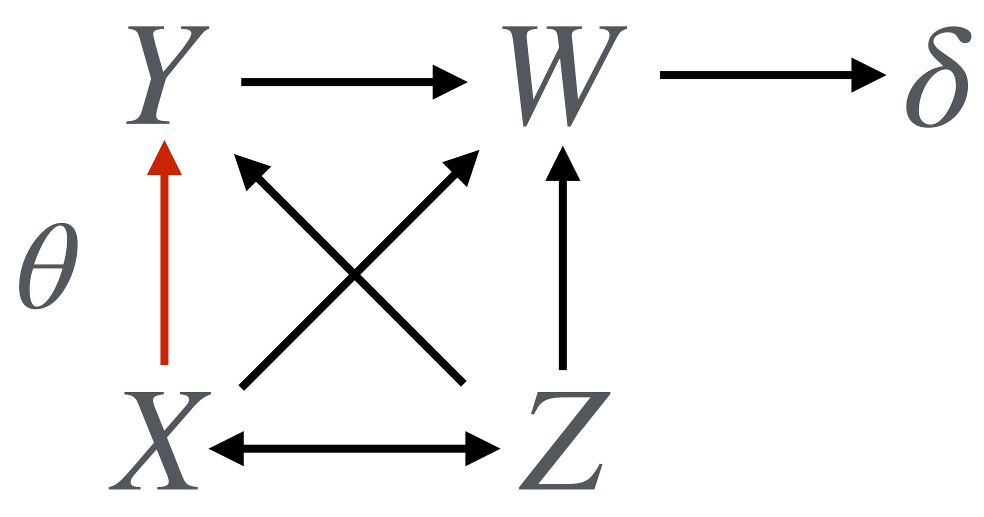


Key Idea: Regard W as a Covariate

- $W^{-1} = P(\delta = 1 \mid X, Y, Z, W)$ is a probability (response probability)

- However, **we treat W as a covariate** and construct a semiparametric model

$$
\begin{aligned}
&f(x, y, z, w \mid \delta = 1; \theta, \eta_1, \eta_2, \eta_3) \\
&= \frac{P(\delta = 1 \mid x, y, z, w) f(z, w \mid x, y; \eta_1) f(y \mid x; \theta, \eta_3) f(x; \eta_2)}{\int P(\delta = 1 \mid x, y, z, w) f(z, w \mid x, y; \eta_1) f(y \mid x; \theta, \eta_3) f(x; \eta_2) \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}w} \\
&= \frac{w^{-1} f(z, w \mid x, y; \eta_1) f(y \mid x; \theta, \eta_3) f(x; \eta_2)}{\int w^{-1} f(z, w \mid x, y; \eta_1) f(y \mid x; \theta, \eta_3) f(x; \eta_2) \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}w}
\end{aligned}
$$



  - η_1, η_2, η_3: infinite dimensional nuisance parameters
  - NOTE: If our interest is estimating outcome model $f(y \mid x; \theta)$, then $f(y \mid x; \theta) = f(y \mid x; \theta, \eta_3)$

- **Goal**: Estimate θ that is not affected by η_1, η_2, η_3

Lemma: Rotnitzky and Robins (1997, Stat. Med.)

Lemma 1. When N is known

The efficient score S_{eff} is given by

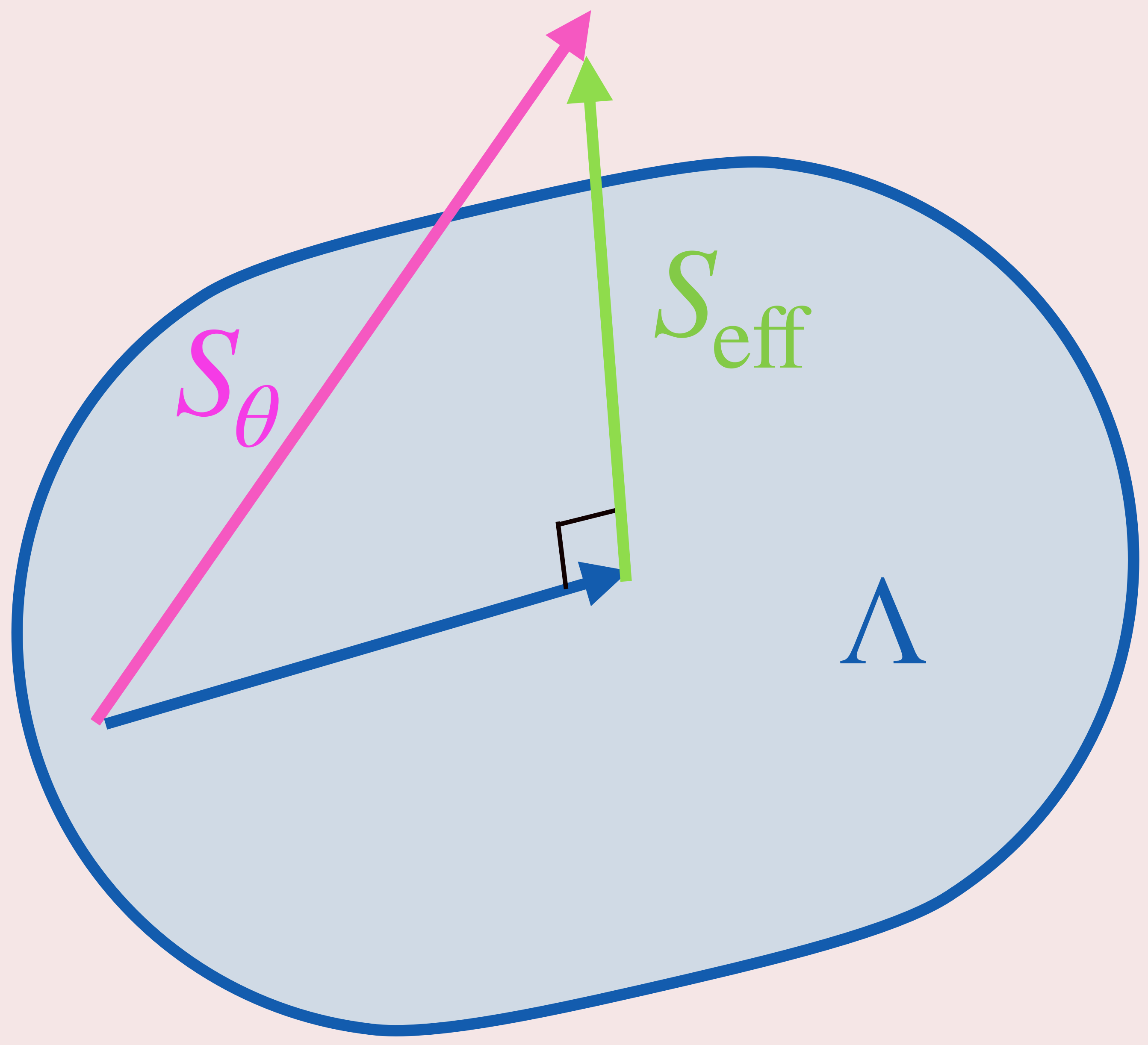
$$S_{\text{eff}} = \delta W D^*_{\text{eff}} + (1 - \delta W)\frac{E\{(W-1)D^*_{\text{eff}}\}}{E(W-1)},$$

$\delta W D^*_{\text{eff}}$ = **IPW**

$(1 - \delta W)\frac{E\{(W-1)D^*_{\text{eff}}\}}{E(W-1)}$ = **Augmented term**

where $D^*_{\text{eff}} \in \Lambda^{F,\perp}$ is the unique solution to

$$\Pi\left(W D^*_{\text{eff}} - (W-1)\frac{E\{(W-1)D^*_{\text{eff}}\}}{E(W-1)} \,\middle|\, \Lambda^{F,\perp} \right) = S^F_{\text{eff}}$$



Then, the semiparametric efficiency bound for θ is $\{E(S_{\text{eff}}^{\otimes 2})\}^{-1}$

Target Parameter

1. Z-estimator: Solution to $E\{U(X, Y; \theta)\} = 0$

$$\theta = E(Y) \ \Rightarrow \ U(X, Y; \theta) = \theta - Y$$

2. Regression parameter: $\mu(X; \theta) = E(Y \mid X)$

3. Outcome model: $f(Y \mid X; \theta)$

Semiparametric Efficiency Bound for θ with partially observed X

Theorem 1. When N is known

The efficient score for θ is

$$S_{\text{eff}} = \delta W D^*_{\text{eff}} + (1 - \delta W) c^*_{\text{eff}} \,,$$

|| IPW || Augmented term

where D^*_{eff} and c^*_{eff} are different according to the target parameters.

The semiparametric efficiency bound for θ is $\{E(S_{\text{eff}}^{\otimes 2})\}^{-1}$

$S_{\text{eff}} = \delta W D^*_{\text{eff}} + (1 - \delta W) c^*_{\text{eff}}$

$$\bar{\pi} = \bar{\pi}(x, y) = \frac{1}{E(W \mid x, y)}$$

(i) $E\{U(X, Y; \theta)\} = 0$:

$$D^*_{\text{eff}} = U(\theta), \qquad c^*_{\text{eff}} = \frac{E\{(W-1)U(\theta)\}}{E(W-1)}.$$

(ii) $\mu(x; \theta) = E(Y \mid x)$

$$D^*_{\text{eff}} = A^*_{\text{eff}}(X)\{\underbrace{Y - \mu(X; \theta)}_{\varepsilon}\}, \qquad c^*_{\text{eff}} = \frac{E\left[\frac{E(W\varepsilon \mid X)}{E(W\varepsilon^2 \mid X)} \frac{\partial}{\partial\theta}\mu(X; \theta)\right]}{E\left[E(W-1) - \frac{\{E(W\varepsilon \mid X)\}^2}{E(W\varepsilon^2 \mid X)}\right]},$$

where

$$A^*_{\text{eff}}(x) = \frac{1}{E(W\varepsilon^2 \mid x)}\left[E(W\varepsilon \mid x)c^*_{\text{eff}} + \frac{\partial}{\partial\theta}\mu(x; \theta)\right]$$

(iii) Outcome model $f(y \mid x; \theta)$:

$$\bar{\pi} = \bar{\pi}(x, y) = \frac{1}{E(W \mid x, y)}$$

$$D^*_{\text{eff}} = \bar{\pi} \left\{ S_\theta - \frac{E(\bar{\pi} S_\theta \mid x)}{E(\bar{\pi} \mid x)} \right\} + \left(1 - \frac{\bar{\pi}}{E(\bar{\pi} \mid x)} \right) c^*_{\text{eff}},$$

$$S_\theta = S_\theta(x, y) = \frac{\log f(y \mid x; \theta)}{\partial \theta}$$

$$c^*_{\text{eff}} = \frac{E\left\{ \frac{E(\bar{\pi} S_\theta \mid X)}{E(\bar{\pi} \mid X)} \right\}}{1 - E\left[\frac{1}{E(\bar{\pi} \mid X)} \right]}$$

$\bar{\pi}(x, y)$

and its conditional expectation

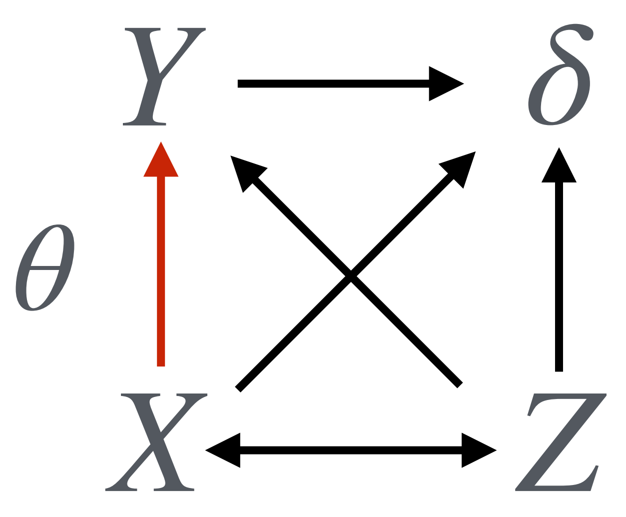
$E(\bar{\pi} \mid x)$ and $E(\bar{\pi} S_\theta \mid x)$

are unknown functions

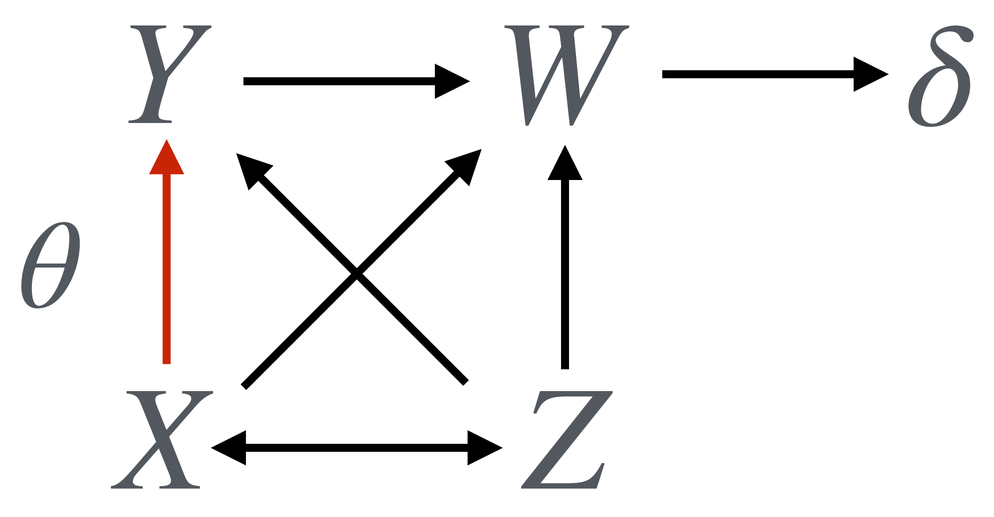
Remark. Z is Unnecessary

- Information of Z does NOT affect efficiency of θ at all
  - In missing data analysis, all the covariates that affect δ are required to be observed
  - However, in this case, observing W is enough to explain δ

- We do NOT need to sample Z even if it has an effect on W



Usual NMAR



Informative sampling

Example. Adaptive Estimator for $E(Y)$

- Estimating Equation: $S_{\text{eff}} = \delta W D^*_{\text{eff}} + (1-\delta W)c^*_{\text{eff}}, \quad U(\theta) = \theta - Y$

$$S_{\text{eff}}(\theta) = \sum_{i=1}^{N} \left\{ \delta_i W_i(\theta - Y_i) + (1-\delta_i W_i)\frac{E\{(W-1)(\theta - Y)\}}{E(W-1)} \right\} = 0$$

$$\Rightarrow \quad \hat{\theta} = \frac{1}{N}\sum_{i=1}^{N} \left\{ \delta_i W_i Y_i + \left(1-\delta_i W_i\right) \frac{E\{(W-1)Y\}}{E(W-1)} \right\}$$

Unknown value

$$\frac{E\{(W-1)Y\}}{E(W-1)} = \frac{E_1\{W(W-1)Y\}}{E_1(W(W-1))} \approx \frac{\sum_{\delta_j=1} W_j(W_j-1)Y_j}{\sum_{\delta_j=1} W_j(W_j-1)}$$

Estimator

Working Models

- Consider an adaptive estimator for (c) $f(y \mid x; \theta)$

- The optimal estimating equation involves estimation of unknown functions:

1. $\bar{\pi}(x, y) = \{E(W \mid x, y)\}^{-1}$

   We give a reasonable model later.

2. $E(\bar{\pi} \mid x) = \int \bar{\pi}(x, y) f(y \mid x; \theta) \mathrm{d}y$ and $E(\bar{\pi} S_\theta \mid x)$

   Because θ is estimable with the Horvitz-Thompson estimator (say, $\hat{\theta}_{\mathrm{HT}}$), this function can be computed by

$$\hat{E}_{\mathrm{HT}}(\bar{\pi} \mid x) = \int \bar{\pi}(x, y) f(y \mid x; \hat{\theta}_{\mathrm{HT}}) \mathrm{d}y$$

Parametric Model on W –1/2–

- $X \sim \text{Beta}(\alpha, \beta) \;\Leftrightarrow\; 1 - X \sim \text{Beta}(\beta, \alpha) \Leftrightarrow \dfrac{1 - X}{X} \sim \text{Beta}'(\beta, \alpha)$

- Assume that $W^{-1} \mid (x, y) \sim \text{Beta}(m(x, y)\phi, \{1 - m(x, y)\}\phi)$
  - W^{-1} take values on $(0,\ 1)$
  - $E(W^{-1} \mid x, y) = m(x, y),\ V(W^{-1} \mid x, y) = \dfrac{m(x, y)\{1 + m(x, y)\}}{1 + \phi}$ (ϕ: precision parameter)
  - This is essentially same as the beta regression model (Ferrari and Chibari-Neto, 2004, J. Appl. Stat.)

- Thus, $O := W - 1 = \dfrac{1 - W^{-1}}{W^{-1}} \sim \text{Beta}'(\{1 - m(x, y)\}\phi,\ m(x, y)\phi)$

Parametric Model on W –2/2–

- Distribution on $O \mid (x, y, \delta = 1)$ $\qquad (W = O + 1)$

$$f_1(o \mid x, y) \propto f(o \mid x, y) P(\delta = 1 \mid x, y, o) = f(o \mid x, y) \frac{1}{1+o}$$

$$= o^{\{1-m(x,y)\}\phi - 1}(1+o)^{-\phi} \cdot \frac{1}{1+o}$$

$$\Rightarrow \quad O \mid (x, y, \delta = 1) \sim \mathrm{Beta}'(\{1 - m(x, y)\}\phi, m(x, y)\phi + 1)$$

- By using a property of the beta prime distribution,

$$E_1(W \mid x, y) = 1 + E_1(O \mid x, y) = \frac{1}{m(x, y)};$$

$$E(W \mid x, y) = 1 + E(O \mid x, y) = \frac{\phi - 1}{m(x, y)\phi - 1}$$

Parametric Model on W

Proposition 1.

$$E(W^{-1} \mid x, y) = m(x, y),\ V(W^{-1} \mid x, y) = \frac{m(x, y)\{1 + m(x, y)\}}{1 + \phi}$$

$\Uparrow$

Assume that $W^{-1} \mid (x, y) \sim \text{Beta}(m(x, y)\phi, \{1 - m(x, y)\}\phi)$.

Then, $W - 1 =: O \mid (x, y) \sim \text{Beta}'(\{1 - m(x, y)\}\phi, m(x, y)\phi)$ and

$$O \mid (x, y, \delta = 1) \sim \text{Beta}'(\{1 - m(x, y)\}\phi, m(x, y)\phi + 1)$$

- The assumption is essentially same as the beta regression model (Ferrari and Chibari-Neto, 2004, J. Appl. Stat.)

- By using the properties of beta prime distribution, we have

$$E_1(W \mid x, y) = 1 + E_1(O \mid x, y) = \frac{1}{m(x, y)};$$

$$E(W \mid x, y) = 1 + E(O \mid x, y) = \frac{\phi - 1}{m(x, y)\phi - 1}$$

Proposed Adaptive Estimator for (c) $f(y \mid x; \theta)$

1. Assume a parametric model on $m(x, y)$, e.g.

$$W^{-1} \mid (x, y) \sim \text{Beta}(m\phi, (1 - m)\phi)$$

$$m(x, y; \beta) = \frac{\exp(\beta_0 + \beta_1 x + \beta_2 y)}{1 + \exp(\beta_0 + \beta_1 x + \beta_2 y)}$$

2. Estimate (ϕ, β) by ML based on the likelihood on $f_1(o \mid x, y)$ (beta prime distribution)

3. Let $\bar{\pi}(x, y; \hat{\beta}, \hat{\phi}) = \dfrac{m(x, y; \hat{\beta})\hat{\phi} - 1}{\hat{\phi} - 1}$

4. Solve the following estimating equation w.r.t. θ (say, $\hat{\theta}_{\text{eff}}$):

$$S_{\text{eff}}(\theta, \hat{\alpha}) := \frac{1}{n} \sum_{i=1}^{n} \left\{ \delta_i W_i \hat{D}^*_{\text{eff}}(X_i, Y_i; \theta, \hat{\alpha}) + (1 - \delta_i W_i)\hat{c}^*_{\text{eff}}(\hat{\alpha}) \right\},$$

where $\hat{\alpha} = (\hat{\beta}^\top, \hat{\phi}, \hat{\theta}_{\text{HT}}^\top)^\top$ and $\hat{D}^*_{\text{eff}}(\theta, \hat{\alpha})$ and $\hat{c}^*_{\text{eff}}(\hat{\alpha})$ are obtained by replacing the unknown functions with the estimated ones.

Efficient Score When N is Unknown

- The efficient score when N is unknown is obtained by letting c^*_{eff} be 0

- For example, if the regression model is of our interest,

$$S_{\text{eff}} = \delta W D^*_{\text{eff}} + (1 - \delta W) \times 0,$$

where $D^*_{\text{eff}} = A^*_{\text{eff}}(X)\{Y - \mu(X;\theta)\}$ and $A^*_{\text{eff}}(x) = \dfrac{1}{E(W\varepsilon^2 \mid x)} \dfrac{\partial}{\partial \theta} \mu(x;\theta)$

This is exactly same as the result of Kim and Skinner (2013, Biometrika)

Summary of Efficient Score

$$S_{\text{eff}} = \delta W D^*_{\text{eff}} + (1 - \delta W) c^*_{\text{eff}}$$

Information		Target parameter θ			
N	X	Z-estimator	Regression	Outcome	
Known	Partial	✔	✔	✔	→ c^*_{eff}: constant
Unknown	Partial	✔	Kim and Skinner (2013, Biometrika)	✔	→ $c^*_{\text{eff}} \equiv 0$
Known	Complete	✔	✔	✔	→ c^*_{eff}: function of x

I focused on this part in this talk

Extension to Strata Mixed Model

- If the sampling mechanism is stratified sampling, it would be reasonable to assume that W^{-1} follows a beta distribution in each stratum h, e.g.

$$W^{-1} \mid (x, y, H = h) \sim \mathrm{Beta}(m_h(x, y)\phi_h, \{1 - m_h(x, y)\}\phi_h)$$

- However, we need an additional model on $P(H = h \mid x, y)$ such as the multinomial logit model
  - The parameters are computable by the EM algorithm

- We can compute $E(W \mid x, y)$ and $E_1(W \mid x, y)$ analogously

Large Sample Property of Proposed Estimator

Theorem 2.

Under some regularity conditions, $\hat{\theta}_{\text{eff}}$ has the following two properties:

(i) if all the working models are correct, $\hat{\theta}_{\text{eff}}$ attains the semiparametric efficiency bound;

(ii) even if all the working models are misspecified, $\hat{\theta}_{\text{eff}}$ has consistency and asymptotic normality. Let α be the parameter of the working models and $\tilde{\alpha}$ be the probability limit of α. Then, the asymptotic variance of $\hat{\theta}_{\text{eff}}$ is given by

$$V(\hat{\theta}_{\text{eff}}) = E\left\{\frac{\partial S_{\text{eff}}(\tilde{\alpha}, \theta^*)}{\partial \theta^\top}\right\}^{-1} E(S_{\text{eff}}^{\otimes 2}(\tilde{\alpha}, \theta^*)) E\left\{\frac{\partial S_{\text{eff}}(\tilde{\alpha}, \theta^*)}{\partial \theta^\top}\right\}^{-1}$$

- Property (ii) insists robustness of $\hat{\theta}_{\text{eff}}$ for model misspecification
- The asymptotic variance is independent of that of $\tilde{\alpha}$
- Model on $m(x, y)$ can be nonparametric

Semi- and Non-parametric Working Model

- Semparametric working model
  - We may keep assuming a beta regression, but with a nonparametric model on $m(x, y)$
- Nonparametric working model
  - By nonparametrically estimating $E_1(W \mid x, y)$ and $E_1(W^2 \mid x, y)$, we can estimate

$$\bar{\pi}(x, y) = \frac{1}{E(W \mid x, y)} = \frac{E_1(W \mid x, y)}{E_1(W^2 \mid x, y)}.$$

- We believe that we can show that estimators with above working models are also valid, but we have not finished to prove yet.

Contents

- Introduction
- Proposed Estimator
- Simulation
- Real Data Analysis

Numerical Study　Setup –1/2–

- Setup:
  - $X \sim N\left(0, \dfrac{1}{\sqrt{2}^2}\right)$, $Z \sim N\left(0, \dfrac{1}{\sqrt{2}^2}\right)$, $Y \mid (x, z) \sim N\left(x - z, \dfrac{1}{\sqrt{2}^2}\right)$
  - $W^{-1} \sim \mathrm{Beta}(m(x,y)\phi, \{1 - m(x,y)\}\phi)$ and $\phi = 2{,}500$
  - $\delta \mid w \sim \mathrm{Binom}(w^{-1})$
  - $N = 5{,}000$: size of a population
  - $B = 1{,}000$: number of iteration
- Model: $Y \mid x \sim N(a + bx, \sigma^2)$
  Target parameter $\theta = (a, b, \sigma^2)^\top$; True value $\theta^* = (0, 1, 1)^\top$

Numerical Study　Setup –2/2–

- Scenarios for $\mu(x, y)$: $n \approx 200$ in all cases

  S1. (No dependency) $\text{logit}\{m(x, y)\} = -3.2$

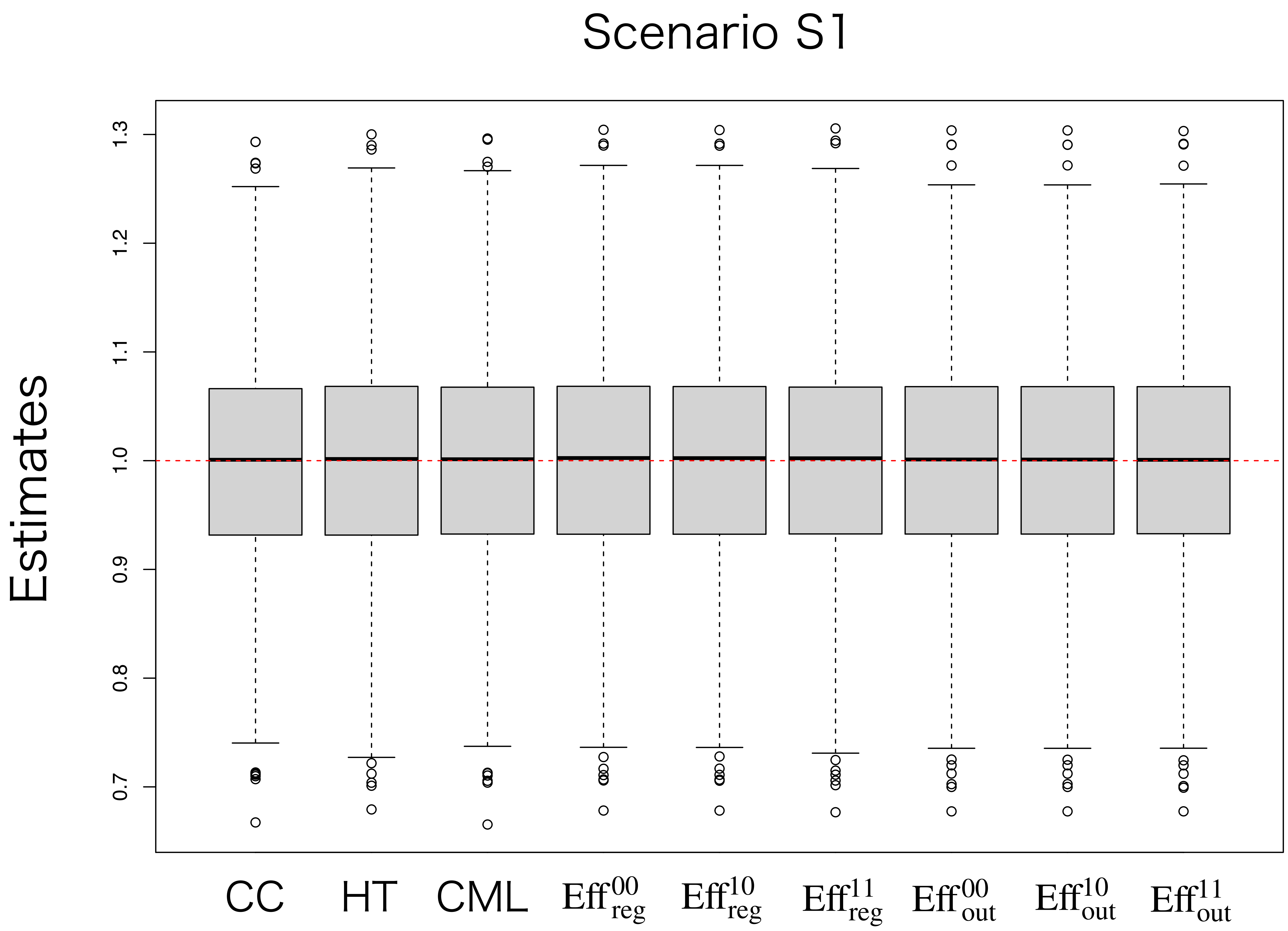
  S2. (Dependency) $\text{logit}\{m(x, y)\} = -3.4 + 0.3x + 0.5y$

  S3. (Misspecified) $\text{logit}\{m(x, y)\} = -3.4 + 0.25x + 0.25z + 0.1y^2$

- Parametric model on $m(x, y)$: $\text{logit}\{m(x, y)\} = \alpha_0 + \alpha_1 x + \alpha_2 y$

- Methods:
  - CC: complete case analysis ($w_i \equiv 1$)
  - HT: Horvitz-Thompson type estimator
  - CML: Conditional Maximum Likelihood
  - Eff_{reg}, Eff_{out}: Proposed estimator
    - reg: adaptive estimator for **reg**ression model
    - out: adaptive estimator for **out**come model

Boxplot for $\hat{b}$ in Scenario S1

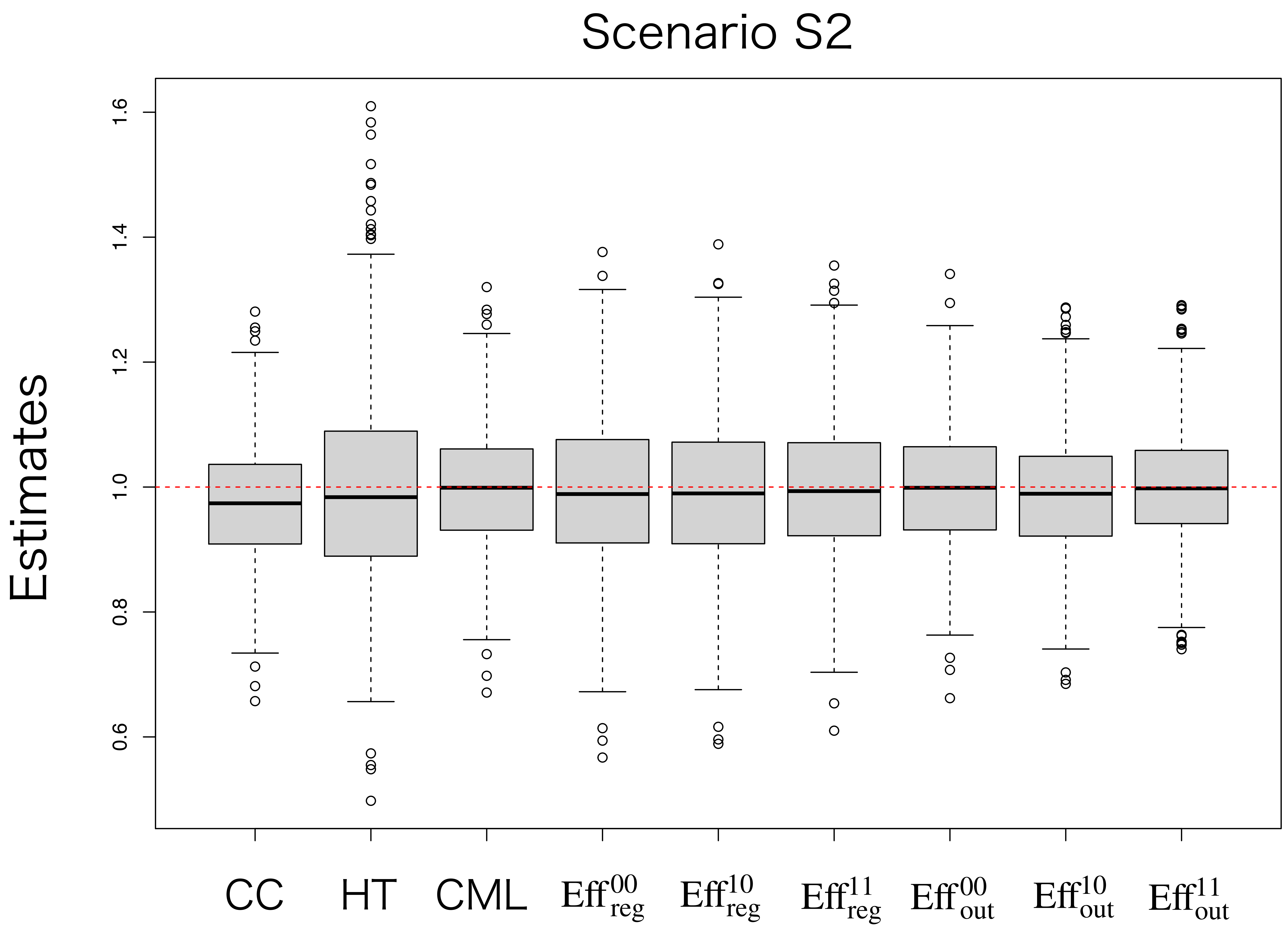


Eff^{ij}

i: N is known? (1/0)

j: X is completely observed?(1/0)

Boxplot for $\hat{b}$ in Scenario S2

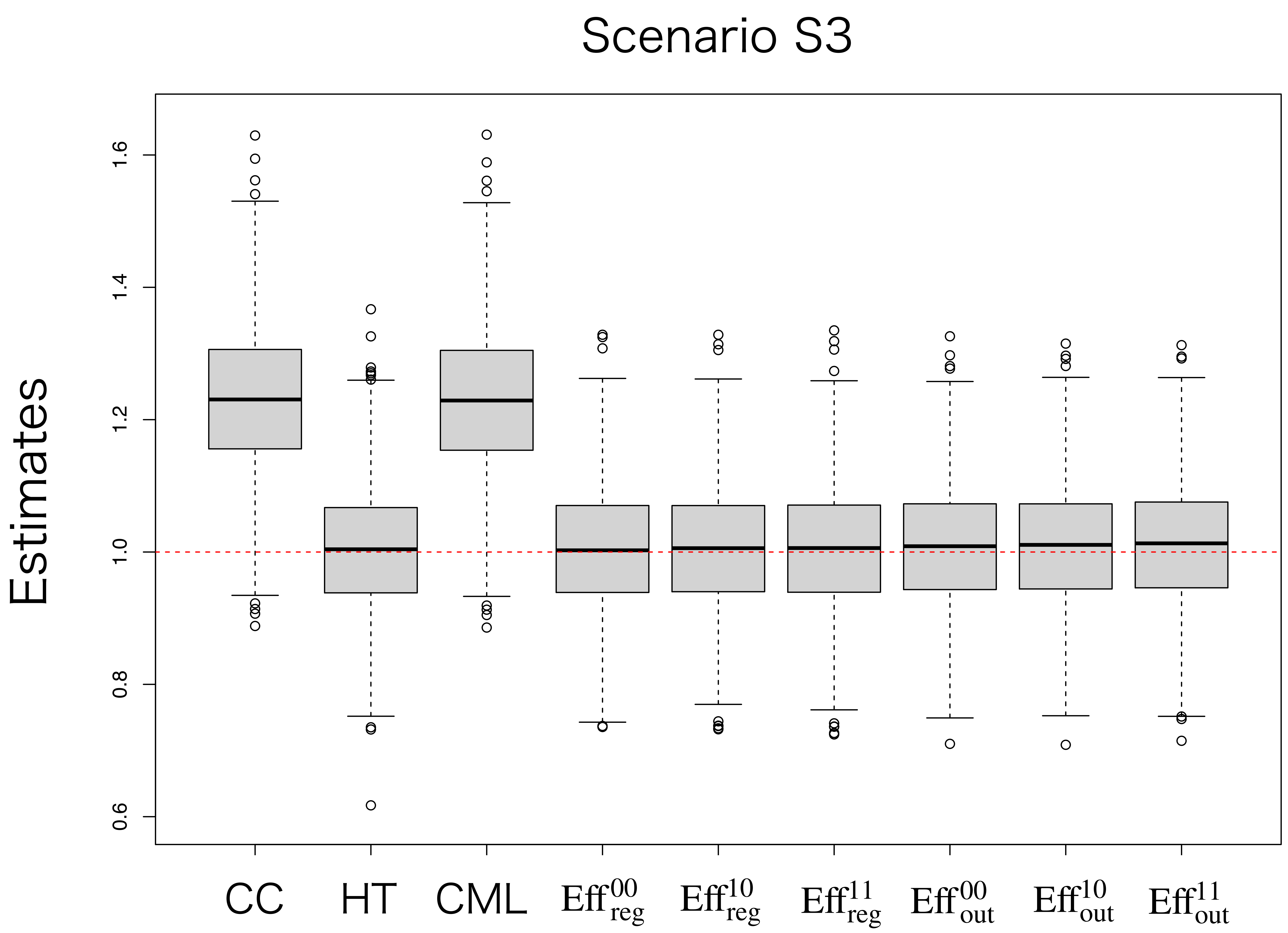


Eff^{ij}

i: N is known? (1/0)

j: X is completely observed?(1/0)

Boxplot for $\hat{b}$ in Scenario S3



Eff^{ij}

i: N is known? (1/0)

j: X is completely observed?(1/0)

Contents

Example: The Canadian Workplace and Employee Survey

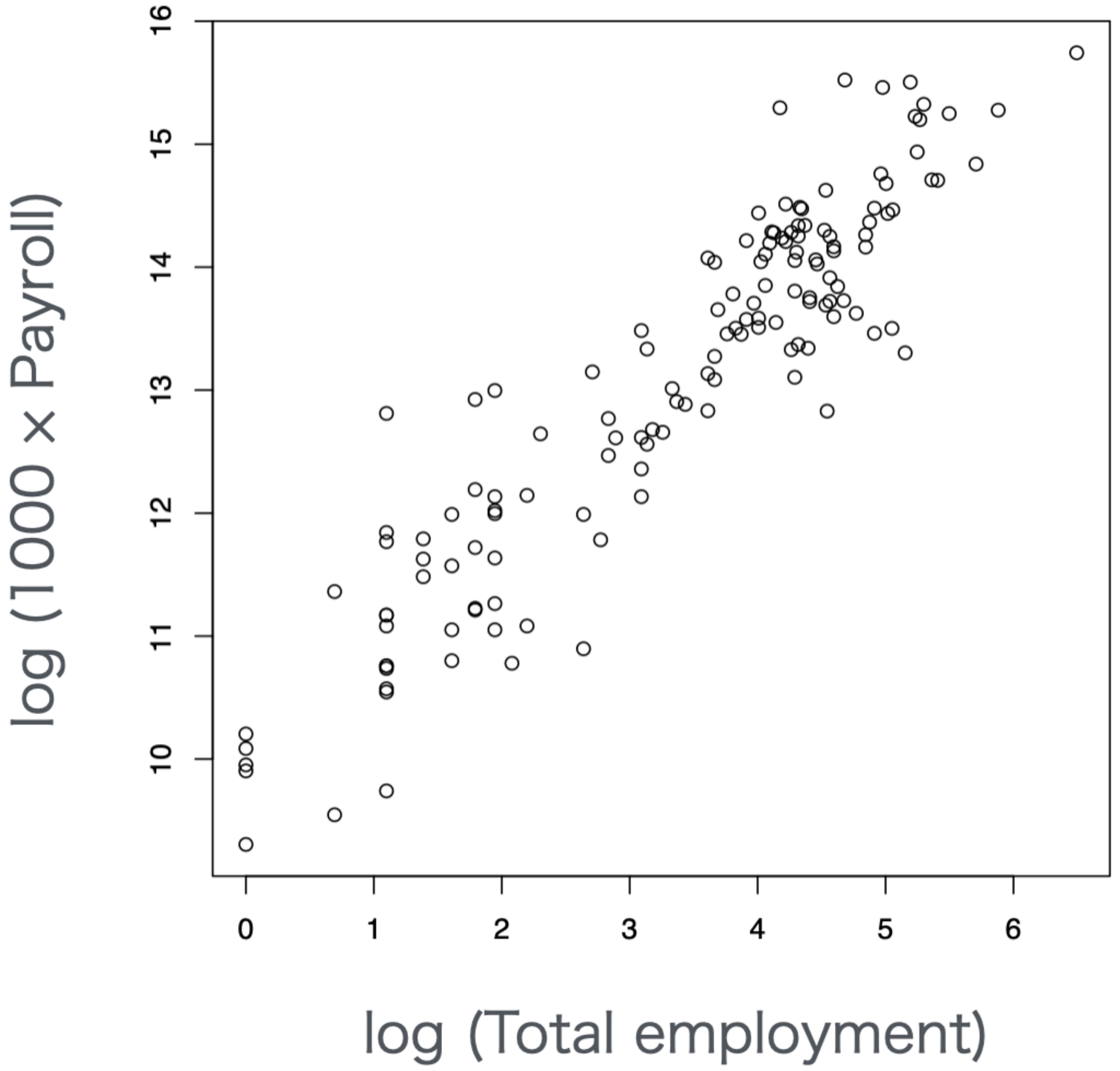
- We want to know the relationship between Payroll (Y) and total Employment (X)

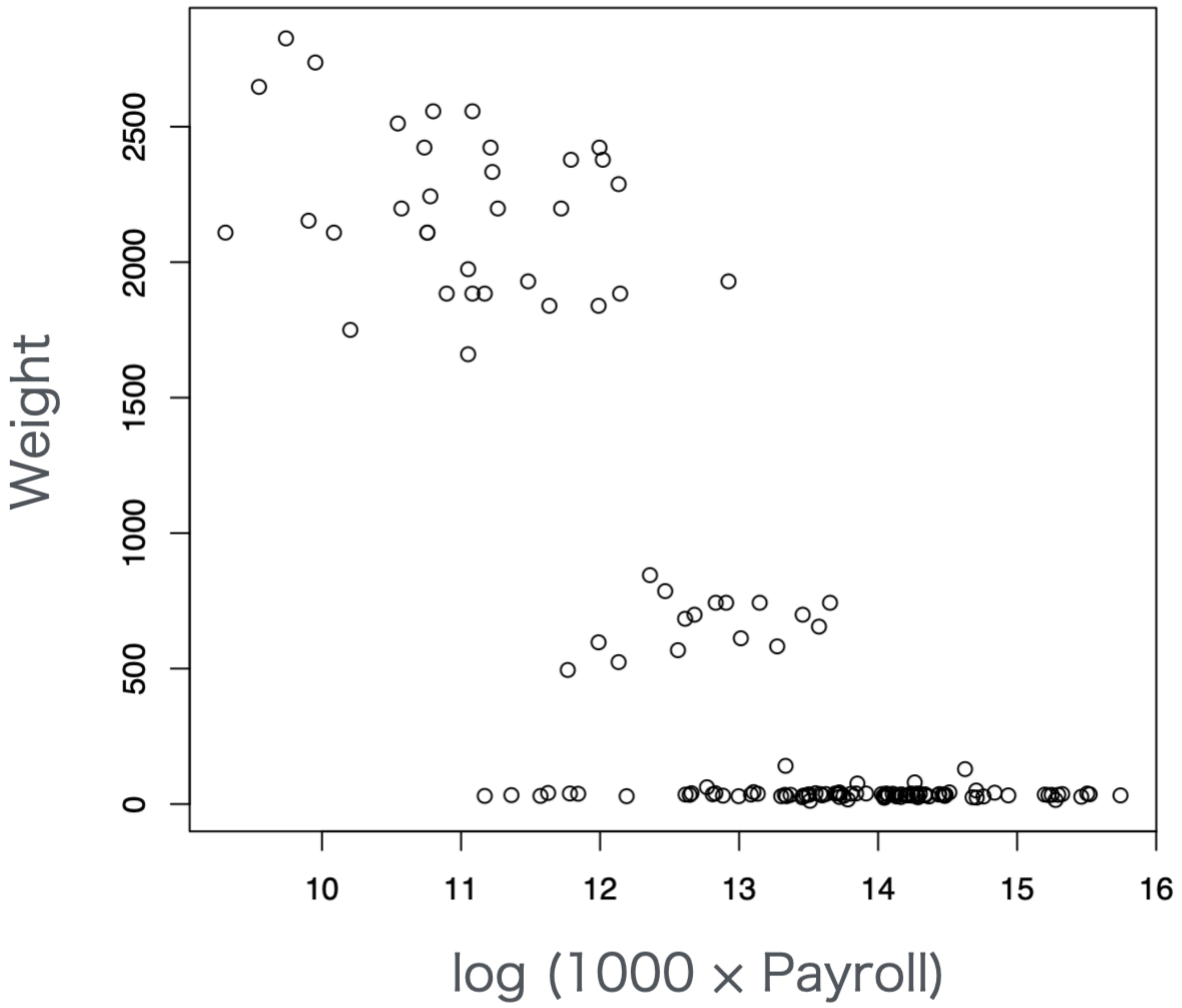
- Size of population (N): 2029 workplaces

- Sampled size (n): 142 workplaces
  - Stratified sampling (3 strata)
    + simple random sampling
      with nonresponse adjustment

- Model:

$$Y \mid X = x \sim N(a + bx, \sigma^2), \quad \theta = (a, b, \sigma^2)$$





Working model

- Mean function of $W^{-1} \mid (x, y, H = h)$:

$$m_h(x, y) = \beta_h \ (h = 1,2,3), \text{ where } 0 < \beta_h < 1$$

- Mixture probability of strata:

$$P(H = h \mid x, y; \gamma)$$

$$= \frac{I(h=1) + I(h=2)\exp(\gamma_0^{(1)} + \gamma_1^{(1)} y) + I(h=3)\exp(\gamma_0^{(2)} + \gamma_1^{(2)} y)}{1 + \exp(\gamma_0^{(1)} + \gamma_1^{(1)} y) + \exp(\gamma_0^{(2)} + \gamma_1^{(2)} y)}$$

Estimates for The Canadian Workplace and Employee Survey

Parameter	Methods		
	CC	HT	$\text{Eff}^{11}_{\text{out}}$
$\hat{a}$	13.082	12.889	12.898
	(0.0477)	(0.1140)	(0.0671)
$\hat{b}$	0.907	0.931	0.931
	(0.0327)	(0.0532)	(0.0370)
$\hat{\sigma}^2$	0.316	0.299	0.295
	(0.0428)	(0.2030)	(0.0666)

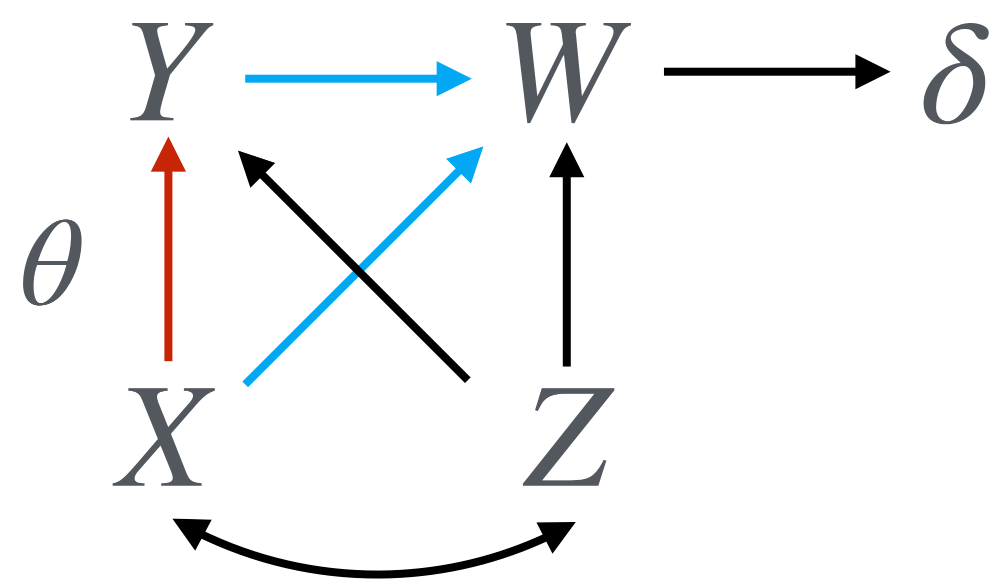
← **estimate**

← **estimated SE**

- Estimates of HT and Eff are very similar
- However, the standard error of Eff is much smaller than HT

Conclusion and Future Works

- In survey sampling, weights are known,
  but the information had NOT been fully utilized



- Our proposed estimator…
  - attains the semiparametric efficiency bound if the working models are correctly specified
  - is robust for misspecification of working models.

- Extension to nonparametric models of the working model


Thank you