Strings: Geometry and Symmetries for Phenomenology

Nana Cabo-Bizet (Universidad de Guanajuato),
Anamaría Font (Universidad Central de Venezuela),
Oscar Loaiza-Brito (Universidad de Guanajuato),
Hans Peter Nilles (Universität Bonn),
Saúl Ramos-Sánchez (Universidad Nacional Autónoma de México),
Fernando Quevedo (Cambridge University)

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1 Overview of the Field

String phenomenology is the study of the possible connections between the mathematical aspects of string theory and model building for particle physics and cosmology. The construction of models that reproduce known features and predict unknown aspects of the Universe requires, among other things, the compactification of six of its spatial dimensions on spaces with very special properties. Over the years, different formal tools have been developed and found to serve to characterize, classify and study these spaces. Those tools belong to broad fields of mathematics, such as complex (differential and algebraic) geometry, group theory, topology and K-theory, among others. Surprisingly, while dealing with physically relevant properties of string constructions, previously unknown mathematics and mathematical relations/dualities have been discovered, and, conversely, advanced mathematical results have turned out to be very important for phenomenology. More recently, also numerical tools as advanced as machine learning have become relevant in this endeavor. This reveals that the study of the physics of string theory requires a constant dialog between specialists on various areas of mathematics and physics.

It is enlightening to review instances of the results of this fruitful interaction. Since the early days, it was established that in order to attain some desirable features of particle physics in four dimensions, it is required that six of the ten dimensions of any of the consistent versions of string theory be compactified on either a six-dimensional Calabi-Yau (CY) manifold [19] or specific orbifold spaces [26, 37, 28]. These studies triggered the discovery of many CY three-folds [43] and a deeper understanding of their properties, including the structure of their moduli spaces and partial knowledge of their metric in the local limits (see e.g. [45]). Simultaneously, the identification of the symmetries in abstract conformal field theories (CFT) allowed the discovery of various dualities that have been and still are the source of research in physics and mathematics. In physics, this led to the discovery of so-called D-branes, the associated fluxes, and ultimately of M-theory, which has triggered the study of 7-manifolds with G_2 holonomy from diverse approaches [17, 25], including complex analysis in the Kovalev construction [27]. Furthermore, the geometrization of the axio-dilaton field in type IIB string theory has led to a non-perturbative description known as F-theory [57]. In this context, the study of the resulting elliptically-fibered CY four-folds used as compactification schemes to arrive at four-dimensional physics as well as the (atomic) classification of (five [38] and) six-dimensional [33] super-conformal field theories based on F-theory are signs of the success of the collaboration of experts in mathematics and physics within the scope of string theory.

Interesting mathematical aspects of CY varieties, such as mirror symmetry [47, 55, 36, 35], can be described in terms of the supersymmetric $(2, 2)$ two-dimensional gauged linear sigma models (GLSM) [58]. In this approach, CY manifolds arise as the supersymmetric vacua of GLSM. Further, GLSM have an infrared fixed point which is expected to be the exact world-sheet CFT description of string theory on a CY manifold. Mirror symmetry for complete intersection Calabi-Yau (CICY) manifolds has been understood in this context as an Abelian T-duality. However, the non-CICYs, such as Pfaffian and determinantal varieties, which arise from GLSM with non-Abelian gauge groups have still to be explored [40], perhaps by applying dualities. As mathematicians have observed, non-CICY manifolds could yield by far the biggest class of CY manifolds. Moreover, since models with $(0, 2)$ supersymmetry give rise to the heterotic string, future attention to these schemes is foreseeable (and welcome).

These tools have been intensively applied to string phenomenology. In the context of weakly coupled heterotic string theories, CY manifolds and orbifold compactifications yield a large set of special models whose emerging particle physics with (and without) supersymmetry is compatible with most observations, and leads to plausible explanations for some of the extant open questions of the Standard Model (see e.g. [49] and references therein). A similar result is found in the scope of the type II theories where, besides the choice of a CY manifold as compactification, orientifolds [7] and a suitable set of intersecting D-branes are used [16]. Furthermore, in the presence of fluxes that allow for the stabilization of the moduli, it is necessary to consider the back-reaction on the internal space that leads to a generalization of the geometry and topology relevant for physics in four-dimensions [30]. The resulting meta-stable vacua have the potential to explain the current structure of our universe, including its curvature, the observed abundances, some early-stage processes such as inflation and reheating, and even perhaps the origin of the big bang as the result of colliding branes [13]. Recently, it has been pointed out that this cosmology could produce some of the features of primordial gravitational waves, which are a focus of current observations. On the other hand, also local F-theory models with some qualities of grand unified theories or similar to the supersymmetric version of the Standard Model have been found [15]. These models are of interest as some estimations suggest that F-theory may be able to identify the largest set of string compactifications [56]. In addition, further compactifications of 6D (5D)superconformal field theories obtained in F-theory, can lead to conceptual insights into effective field theories which could be relevant for phenomenology.

2 Recent Developments and Open Problems

Despite the successes mentioned above, string phenomenology still faces several challenges towards testable predictions. For example, recently a link has been identified between target-space modular symmetries, natural to toroidal orbifold compactifications, and flavor and neutrino physics [14]. Although this sets an ideal scenario for phenomenology, the concrete structure of these symmetries in string theory, including their breakdown, is still a matter of ongoing research. Since this physics is currently probed in laboratories and almost any symmetry can be imposed in bottom-up scenarios, aiming at an ultraviolet (UV) completion of observable physics stresses the importance of understanding the flavor structure of string models. Remarkably, this structure may also explain the origin of CP violation [50] and the symmetries that stabilize dark matter [54]. These considerations become now a key ingredient to the solution of the long-standing issue of determining the coupling strengths for the interactions in the effective field theory of string compactifications [23, 3, 42, 18]. Similarly, in F-theory the source and properties of flavor and U(1) global symmetries must be addressed.

Another problem to solve is the stabilization of moduli in string vacua [12]. As mentioned earlier, this could be achieved in type II theories by incorporating (geometric or non-geometric) fluxes in the compact dimensions. In this context, the obtained meta-stable vacua display properties of a universe where the breakdown of supersymmetry and single or multi-field inflation can occur, leaving signals (albeit weak) that could be tested against observations. Non-geometric fluxes, i.e. T-dual manifest versions of type II flux compactifications, can be advantageous as they allow to treat all moduli on the same footing [6, 53] leading to interesting vacua. However, problems arise when one focuses on open string moduli or when one studies the back-reaction from the dynamics of the stabilized fields on the geometry of space-time. Furthermore, it is known that one of the most difficult tasks is finding meta-stable string vacua with, additionally, both a proper cosmological constant and admissible particle physics. On the other hand, in the context of heterotic constructions and F-theory models, it is so far not even fully established whether meta-stable vacua with or without supersymmetry exist. Some of the aspects to deal with these issues, that need the collaboration of experts of all these topics, include the relations between moduli stabilization and number theory [21, 20] and symmetries of the underlying CFT of string theory, which e.g. could help answer the question of whether moduli stabilization leading to dS vacua is possible in the heterotic string [44].

In addition, moduli stabilization is closely related to the characteristic landscape of string vacua. Identifying the string model with stabilized moduli that best describes our universe among the enormously vast number of string vacua requires a set of computational tools beyond the traditional ones. The recent application of machine learning and data analysis techniques could improve the situation in the coming years. In particular, promising results have already been obtained in studies of the geometry of F-theory [22], the phenomenology of heterotic compactifications [48] and the properties of field and string theory vacua [31].

String models are also fruitful sources for the use of more general mathematics and for the construction of extended models in which it is possible to ask whether some expected effective features, such as inflation or a de Sitter vacuum with adequate cosmological constant, are compatible with the theory at high energies. Within this context, recently, in a series of so-called swampland conjectures, it has been suggested that most of the effective 4D field theories that were thought to stem from string compactifications may be incompatible with an ultraviolet complete theory that includes gravity [11, 51]. Much attention has been paid to the proper formulation and consequences of these conjectures because they could rule out large classes of models and techniques used on string model building, enormously constraining the landscape of promising models and their predictions [52], and even imposing restrictions to bottom-up models of inflation. There are already steps in exploring these conjectures in heterotic strings, type IIB strings and F-Theory. One of the most important goals in this scope is to understand whether string theory could reveal the microscopic origin and thus render a demonstration of the conjectures. The optimistic scenario would most likely allow one to identify the vacua of string theory and, ultimately, to contrast them with observations.

3 Presentations

The workshop's talks and discussions were virtual, shared via Zoom, due to the limitations associated with the current pandemic. The workshop consisted of morning sessions, which accommodated speakers from Asia, Europe and the Americas presenting progress on five different research topics: swampland conjectures, string phenomenology, string cosmology, machine learning and mathematics of string theory. Each day, there were two long plenary talks with plenty of time for questions, and there was a discussion session led by renowned experts of the different topics. Beyond the excellent talks, the discussion sessions provided a fertile arena for exchanging ideas and assessing more dynamically the state-of-the-art of the area, aiming at drawing future venues to tackle the extant questions of the field.

3.1 Swampland conjectures

The talks were presented by Lars Aalsma and Irene Valenzuela. They both explored the impact of studying black hole physics on the understanding of quantum gravity. Aalsma's talk was based on [1, 2]. He explained how the study of extremal black holes can help to understand the origin of (a mild form of) the so-called Weak Gravity Conjecture (WGC), which may contain some information of quantum gravity. Interestingly, the WGC can be reformulated in terms of the stress tensor of extremal black holes by using the Iyer-Wald formalism. This enables one to extend this study to other black holes and, from that, to learn some features of the intrinsic UV nature of gravity.

After discussing the path of the swampland program, Valenzuela showed that the finiteness of the black hole entropy can explain the ideas related to both the WGC and the distance conjecture. More generally, assuming that all amplitudes in an UV complete theory of gravity are finite could well establish bounds on the number of vacua that are consistent with quantum gravity. This result can be found in her work [32].

Miguel Montero made an overview presentation on the progress, challenges and prospects of the swampland program. In his view, some of the most urgent questions can be stated as follows:

- How can we go from the current rigorously proven (but not so interesting) and interesting (but not well proven) conjectures to statements that are both proven and interesting?

- Since supersymmetry has not been observed, what kind of conjectures and consequences can we obtain without demanding supersymmetry?
- What are the bounds (e.g. in their size) for discrete symmetries given a cut-off scale?
- How can we formally prove the statements conjectured about quantum gravity beyond general arguments taken from string theory?
- Do scale-separated anti-de Sitter vacua exist?
- Can we make concrete predictions for low-energy physics in our vacuum?

Perhaps these questions should guide our discussions in the field.

3.2 Particle phenomenology from string compactifications

For particle physics emerging from string theory, Tatsuo Kobayashi and Michael Ratz explored in their talks some of the ideas that are currently being explored. Kobayashi stressed that there are several D-brane models that reproduce many features of observable physics. However, one of the main puzzles is the predicted size of the couplings among the matter fields, which is perhaps the main source of uncertainty in string constructions. He discussed some efforts towards this goal [34, 4] and possible consequences as i) explanation of the origin of minimal flavor violation [41] and ii) the origin of some flavor symmetries of particle physics.

Along similar lines, Ratz stressed the growing relevance that modular invariance is playing within both string and bottom-up model building. And its relevance is only enhanced when put in the context of other many symmetries that arise naturally in string theory, such as discrete and continuous R symmetries, CP and CP-like symmetries (whose differences are crucial for phenomenology) and flavor symmetries which are partly modular and partly geometric. Aiming at predictions, one should also explore the possibility of realizing these symmetries in string models without supersymmetry. Another interesting observation of this talk was the possibility within string theory that we do not yet know the actual number of generations in string models, as strong interactions can dynamically change it.

Ignatios Antoniadis, Luis Ibáñez and Hans Peter Nilles presented their views of this field during the discussion session. They proposed that some of the most important questions could be:

- How can we dynamically stabilize moduli, so that we can avoid conflict with observations and predict couplings among matter?
- What are the properties of supersymmetry breaking and its consequences?
- Can we build an appropriate model of particle physics *and* cosmology based on string compactifications?
- What alternatives does string theory offer to naturalness arguments as solutions to fine-tuning issues?
- How can we include in string phenomenology the version of quantum gravity emerging from string theory?
- Are there already any signs in Nature that favor or contradict string theory?

3.3 String cosmology

String cosmology might offer a rich framework to discover connections between string theory and observations. This is particularly true with two of the most important problems in cosmology: the nature of inflation and dark energy. On these topics, Ivonne Zavala and Michele Cicoli presented their most recent results in their talks. Zavala discussed the realization and consequences of multifield inflation in the context of both string compactifications and supergravity, according to her work [10]. In particular, she emphasized that many (heavy or *fat*) scalars, which are always present in string models, can help to circumvent the usual η problem. Their dynamics is very rich, leading to complicated trajectories in field space that might be phenomenologically useful. Further, the existence of additional gauge fields can also lead to the production of primordial gravitational waves.

Cicoli, on the other hand, presented some of the results that subsequently appeared in [24]. He took as a basis some of the conclusions drawn from the de-Sitter conjecture, which seem to imply an impossibility of inflation or long-lived quintessence as an alternative to a stable de-Sitter vacuum. Considering string models based on type II compactifications (with axions), he showed the challenges to obtain in a controlled fashion inflation and quintessence. In his paper, he shows that it is still possible to arrive at a string scenario with quintessence, improving over the conjectured absence of a stable positive cosmological constant.

Shamit Kachru and Fernando Quevedo in the discussion session presented their ideas concerning the possibilities that string models describe our cosmology both before and after big bang nucleosynthesis. Arguing that a de-Sitter vacuum can be achieved within string theory, they conveyed the following interesting questions to be addressed:

- Which kind of de-Sitter vacua can we find in string models? Is it already well established whether string theory yields quintessence or a stable cosmological constant?
- What is the dynamics of vacua in the string landscape?
- What string inflationary models that match observations are possible and what are their predictions?
- What are the consequences for reheating, dark matter, baryogenesis and similar post-inflationary processes in string constructions?
- Could primordial gravitational waves serve as the best tool to test string theory?

3.4 Machine learning in string theory

The seminars on machine learning in the context of string theory were given by Fabian Ruehle and Damián Mayorga Peña. Both discussed the challenge of determining the metric of the spaces used in string compactifications. Ruehle based his presentation on [8, 46]. He showed that specific neural networks can provide a good approximation of the metric of any CICY and some extensions of CICYs. Interestingly, one application of these metrics is the computation of masses of moduli-dependent string excitations, which opens a bridge with the distance conjecture of the swampland.

Mayorga Peña discussed the metric for the Ricci-flat manifolds K3, Fermat quintic, and Dwork quintic, as in his work [39]. He studied the details of a simple but very useful neural network, emphasizing its symmetries and their advantages.

In the discussion session, Jim Halverson, Vishnu Jejjala and Andre Lukas reviewed the three forms of machine learning and their applications, triumphs and challenges, when applied to string theory. Regarding supervised learning, we have seen its uses on the study of the properties of CICYs, line-bundle cohomology, knot theory and even a machine that may generate new conjectures on the swampland program. Self-supervised learning is more challenging, but is precisely the method on which the works that determine numerically the metric of different spaces are based. Finally, reinforced learning seems to be best suited to explore large classes of string compactifications, build new models, and perhaps identifying the best ones among the plethora of them that can be built.

Some of the open questions in this field are

- On the more theoretical side, how can machines better learn?
- What kind of questions can we answer from this approach?
- Can we embed symmetries in the architecture of a neural network with phenomenological purposes?
- Can we use machine learning to identify the best model(s) that describes both particle physics and cosmology?

3.5 Mathematical aspects of string theory

The final day of the workshop was devoted to questions on the mathematical tools that have been developed to address more applications of string theory. The talks were held by Iñaki García Etxebarria and Savdeep Sethi. García Etxebarria based his presentation on [29, 5, 9]. He discussed how $(d+1)$-dimensional topological field theories (TQFTs) encode the symmetries and anomalies of d-dimensional theories that arise in M-theory. This leads to a deeper understanding of the origin of the symmetries of our d-dimensional theories in terms of the larger TQFTs, allowing for computations that otherwise are not direct. Further, this insight can be applied beyond string theory to strongly coupled systems and diverse dualities.

Sethi described in his talk a strategy that can lead to non-supersymmetric AdS models from type II string theory, where the constraints established in swampland conjectures can be overcome. These models can be achieved by going beyond the (semi-)classical approach that is typically used to study cosmology in string models.

In the discussion session Washington Taylor and Albrecht Klemm addressed some of the aspects that we have learned from mathematical features of string compactifications (gauge groups, matter, family replication, supersymmetry) and how these could be extended to learn more. Some of the relevant questions one may be interested to answer soon and for which there are already some efforts, are:

- Could the consistency of quantum gravity render finite the number of effective field theories arising from string theory?
- Can we obtain the standard model gauge group from larger groups by a non-Higgs mechanism?
- Can (non-resolved) singular Calabi-Yau spaces reveal new aspects of (chiral and non-chiral) matter and moduli stabilization?
- Does the moduli space of Calabi-Yau have finite volume? What would be the relation of this question with the swampland?
- Can we draw some lessons for string phenomenology from arithmetic geometry?
- Is there a possibility to relate the amplitude program to some geometrical aspects of compactifications that may help to get closer to phenomenology?

4 Outcome of the Meeting

Despite the fact that nothing matches a meeting in person to dynamically arrive at scientific results, the main goals of our virtual meeting were met. We aimed at providing an environment in which members of seemingly separate areas could find that their research shares many common interests, and their expertise and perspective can help one another to develop steps towards the resolution of the current problems of string model building.

The extraordinary talks and discussions conveyed a clear overview of the conceptual and calculational progress as well as the unsolved puzzles that we should consider in order to achieve the ambitious goal of bringing string theory a step closer to observations. At the intersection of all the mathematical, numerical and conceptual elements discussed in the meeting, the insight that the different participants shared generated the right atmosphere to realize that our ideas are complementary. The rich discussions opened the possibility of new collaborations and nourished the existing collaborations with new questions and ideas to implement. Furthermore, the young researchers that participated in the workshop profited from the perspectives that experts in the field have gathered and shared with everybody, feeling now compelled to address the urgent questions that we mention above.

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