MULTI-STAGE STOCHASTIC OPTIMIZATION FOR CLEAN ENERGY TRANSITION

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The BIRS-CMO 5 days workshop *Multi-Stage Stochastic Optimization For Clean Energy Transition* stands at the interface between a societal concern and goal — clean energy transition — and mathematical methods — here, stochastic optimization — that can contribute to that goal. This is why, our report[1] starts with two overview sections, namely Sect. 1 on clean energy transition, followed by Sect. 2 on optimization under uncertainty. Quite naturally, the next Sect. 3 deals with the mathematical challenges raised by the clean energy transition. Then, in Sect. 4, we highlight the main contributions of the workshop presentations regarding these mathematical challenges. Scientific advances are outlined in Sect. 5. The report concludes with the outcomes of the meeting in Sect. 6.

1 Overview of the Field: Clean Energy Transition

By clean energy transition, we mean the changes in energy systems that are driven by the necessity to reduce CO2 emissions. To address these changes — and, later, to provide the context for how they challenge mathematical optimization — we first sketch the fundamentals of energy management and, especially, of electricity.

1.1 The fundamentals of energy management

Whether managed in a centralized or in a decentralized manner, the operation of an electrical system is subject to a few fundamental aspects. We present these key points within the context of a fully integrated management (when generation, distribution and supply are controlled by a unique stakeholder), but they apply when management is decentralized and in the hand of several decision-makers.

1.1.1 Supply-demand balance: a physical and economic necessity

The first mission of a unique (centralized) operator is to ensure the supply-demand balance. By using both physical and financial means (supply), this operator must provide the electricity needed by its customers (demand) at each time instant. In addition to the service provided to the end electricity users, this balance must also be preserved for crucial reasons related to the electrical network operations (frequency control).

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Both supply and demand quantities must be balanced at each instant due to the current difficulty of storing energy at reasonable cost. This must be anticipated at different time scales from the long-term, when planning the evolution of power generation capacities, to the short-term, when adjusting in real time the generation planning of the different units and also eventually using different financial electricity contracts.

1.1.2 The weather and the market: conditioning fluctuations

Ensuring the supply-demand balance is not an easy task. One of the main reasons for that is the stochastic nature of the main determinants of both supply and demand.

As a first determinant, the weather directly, and significantly, impacts both supply and demand. It conditions the need for heating, but also water flows (in dams), and primary energy resources of renewable units (sun and wind). A huge amount of research work is dedicated to make the link between weather forecast methods and the variables of the electrical system which strongly rely on it. In particular, the prediction of renewable energy generation is a key topic [23, 58]. Furthermore, these studies have to be now conducted at different temporal and geographical scales. Whereas, at a global scale, one faces a variability that is rather smooth (as the aggregate sum of many small random effects), this is no longer the case at a local scale; there, sun and wind can dramatically change at short time scales, and one faces intermittency.

The outage of production plants is another crucial stochastic aspect in energy management. In addition with these physical uncertainties, the energy market fluctuations must also be taken into account. Many studies model the formation of energy market prices [13, 31], introducing a stochastic component.

1.1.3 Protection of the environment

The balance of supply and demand must be ensured while taking into account a certain number of rules to protect the environment and to combat climate change. For the electricity producers, such rules mainly consists in the following: limiting (or reducing) the greenhouse gas emissions; adapting to the rapid development of renewable energies; evaluating, on the demand-side, the modifications induced by the new buildings regulation (High Environment Quality labels).

1.1.4 Energy management in this context

Energy management consists in making decisions to optimally manage both physical and financial assets, while providing the services contracted with the customers and respecting current legislative and commercial constraints. As randomness occurs in weather, energy demand and markets, the risk must also be taken into account. Furthermore, energy management decisions have to be taken at different horizons, from long-term investment strategic choices, to mid/short-term operation planning for the production units and to very short-term supply-demand balance adjustements.

1.2 New decentralized energy systems

France is an example of a move from a vertically integrated management towards a horizontal decentralized one. With the construction of the European electricity markets, opening-up to competition, generation, transmission and distribution activities have been — functionaly then legally — separated in France. This lead to the creation of both a Transmission Network Operator (TNO) and a Distribution Network Operator (DNO). These new players are respectively responsible of providing an equal access — in an objective and transparent manner — to the transmission and distribution networks. This allows the new power system players to access to the services provided by the electrical network.

This move naturally leads to a decentralized framework, by distinguishing different players — producers, TNO, DNO, suppliers — with different objectives, information available, and possible actions. It induces a need for coordination and information exchange between the different entities involved. Designing good rules for this new "game" will condition the performance of the different services, whose management is now shared between various players. The legislator will have a key role in this context; he directly contributes to the design of electricity markets, and to the definition of the rules for the new services proposed to the customers.

Let us now present the main ingredients of this new decentralized vision of energy management in the electricity system.

1.2.1 The multiplicity of stakeholders in the energy system

As mentioned above, new players have recently entered the electricity system. Indeed, they come to fill the different roles that have opened up, now that the objective and management of production, network management and electricity supply are clearly distinguished. But there is more than these traditional stakeholders.

With the sociological and economic trend of reducing — or at least better understanding and controlling — the energy usages, many new players enter this system through the rise of so-called smart grids, or smart cities. This movement comes hand in hand with the progressive implication of society into the question of environmental protection. This stimulates everyone's involvement, starting from the motivation to get informed about his own consumption. More than being informed, big energy customers are already thinking of making their consumption profile flexible according to individual metrics (electricity bill), or even taking into account (local) constraints of the electricity system (energy losses, asset aging).

The new paradigm of smart grid proposes to enlarge the pool of flexible customers to all households [22]. By optimally scheduling its own load profile, each household can meet the objective of minimum energy payment or maximum comfort [41]. This modifies the traditional relation between energy suppliers and their customers, leading naturally to a competitive framework. City operators are also envisioned to be part of this evolution by optimizing their energy consumption profiles. Smart lighting, smart scheduling of electric bus charging profiles, coordination of the charging decisions of a public system of shared vehicles or a taxi fleet are a few examples of this attempt. By making their consumption flexible, all these new players will directly impact the aggregate energy demand profile. Thus, all these new decision-makers should be integrated in the decentralized vision of the energy system.

1.2.2 The new physical generation means: rapid growth of renewable energies

The multiplicity of stakeholders in the energy system comes with the development of new generation means, which also naturally leads to a decentralized vision.

Here, we remind the main characteristics of renewable energies generation. On the one hand, its variable costs and polluting emissions are (approximately) zero, which makes it particularly appealing in the current evolution. On the other hand, wind and solar production are basically intermittent and hence difficult to forecast. There is a clear need for local management of this production, in coordination with the electricity network [60, 56]. In most cases, local production is thought to be used locally to promote self-production and self-consumption. However, the service provided is not completly local, in the sense that the support of the interconnected network is needed in the event of a punctual high need.

In conjunction with these new physical supply means, the concept of virtual power plant [51] is now emerging. It defines a system integrating several types of physical or virtual power sources, so as to give a reliable overall power supply. For example, it can consist of the aggregation of many renewable energy units in order to mitigate the significant intermittency issue faced by a single unit. Forming a coalition [3], renewable producers can exploit the reduced aggregate power output intermittency to submit less risky offers to an electricity market. A virtual power plant can also integrate flexible electricity users, e.g. with shedding mechanisms.

1.2.3 New electricity consumptions and equipments

Many new electricity appliances also lead to rethink the standard centralized vision of the electricity system. Mobile electricity consumption units and electric vehicles are typical examples of these new consumptions. For electric vehicles, the problem consists in designing good charging policies, if possible in interaction with the constraints observed on the electricity system; when capable of reinjecting electricity to the grid (vehicle-to-grid), electric vehicles can also be considered as a local production unit. Furthermore, affordable residential storage units could help the development of local consumption units, without necessarily being connected to the global electricity network.

1.2.4 New information technologies and capabilities

The envisioned smart grids of the future [35] should include a variety of sensors — providing local measures on the electricity network and on the consumption units — and control chips. By using smart meters, a detailed analysis of the residential consumption profiles will be possible, as well as sending incentive prices to these customers. In particular, this allows designing local energy management strategies to coordinate the consumption decisions of a set of electricity users [41]. These newly available measures can be coupled with the fast-growing potential of learning methods, applied on large sets of data (big data).

1.2.5 A new role for the national electricity network?

In a decentralized system, the national electricity network tends to play the role of an insurance provider: indeed, when local production is not enough, one can turn to the network. However, the electricity system was not conceived with this operational vision in mind, and neither were the current economic signals sent to the consumption units. In a decentralized context, relevant questions arise: with the current electricity network structure, what kind of signals should be sent to the local electricity consumption units to properly guide their electricity network use? What would be the new structure of the electricity distribution (and transmission) network if energy management becomes more and more decentralized?

2 Overview of the Field: Optimization Under Uncertainty

To provide an overview of the field, we have chosen to move from stochastic optimization to game theory, in a series of progressive steps.

2.1 Multi-stage Stochastic Optimization

In mathematical optimization, one tries to minimize (or maximize) a function (criterion) under constraints over the decision variable.

In *optimization under uncertainty*, there are additional unkwown variables that can affect the criterion or the constraints. In a nutshell, *robust optimization* looks for a solution, that is deterministic in the sense that it does not depend on the uncertain variables, and that minimizes the worst-case value of the criterion or the worst-case constraints. In contrast, *stochastic optimization* assumes that uncertain variables are random variables (with known probability distribution); then, one looks for a solution that is itself a random variable (possibly deterministic) and that minimizes the expected value of the criterion under almost sure constraints. From here, stochastic optimization radiates in various directions.

Instead of almost sure constraints, one can turn to *probability constraints*, or mix various types of constraints (in probability, robust, in expectancy, etc.).

One can replace the expected value by any so-called risk measure; one can also drop the assumption that there is a known probability distribution, replace the expected value by a risk measure, and one enters the realm of *risk sensitive optimization*, or *distributionally robust optimization*.

In one-stage optimization under uncertainty, one makes a single deterministic decision, then the uncertain variables materialize. In two-stage optimization under uncertainty, one makes a first deterministic decision, followed by a second one that can take advantage of the observation of the uncertain variables. In multi-stage optimization under uncertainty, the scheme goes on where information unfolds at each time step. We will not speak of the case where decisions are made continuously over time. In the multi-stage setting, a solution of a *multi-stage stochastic optimization problem* must satisfy so-called nonanticipativity constraints, expressing than decisions cannot rely on future outcomes.

In *stochastic team optimization problems*, decision-makers jointly contribute to minimize a common criterion, but do not share the same information. Thus, multi-stage stochastic optimization appears as a special case where decision-makers are aligned along temporal steps with an increasing information structure represented by a filtration. Such team optimization problems can, in turn, be static or dynamic, depending on the specific information structure.

2.2 Stochastic Equilibria and Game Theory

We now move from problems with a single criterion to ones with multiple criteria. When these multiple criteria are under the control of a single decision-maker, this is called *multi-objecive optimization* with the central notion of Pareto equilibrium as solution. When these multiple criteria are under the control of multiple decision-makers, but they share the same decision variable, we have *bargaining problems*. We do not have a clear vision of such frameworks extend as such to the uncertain setting; it could possibly be an opportunity for future research.

Game theory is the domain of multiple players, each with its decision set and with its criterion, but the criterion depends on the decision made by all the players. From here, game theory radiates in various directions that we only sketch.

In *non-cooperative game theory*, the central notion of solution is that of Nash equilibrium, pure or mixed. Time enters into play with Stackelberg equilibrium, sequential games, etc. Stochasticity enters into play with types and Bayesian games. The so-called extensive forms of games covers the case of players making successive decisions depending on time, randomness and other players decisions; this is the domain of *games with information* where refined notions of equilibrium have been developed (sequential equilibrium, subgame perfect equilibrium).

There are many games, like the famous Prisoners' dilemma, in which the Nash equilibria yield very low payoff for the players, relatively to other non equilibrium outcomes, like the ones of Pareto equilibria. In such situations, one would want to transform the game, if possible, to extend the set of equilibria to include better outcomes.

It is the goal of public economics to design incentives to move away from socially undesirable Nash equilibria and to achieve Pareto ones, or socially optimum ones. Under uncertainty, this is *stochastic equilibrium* analysis, where one looks for proper price signals or contracts between players in order to decentralize a Pareto optimum.

More generally, it is the goal of *mechanism design* to analyze proper economic mechanisms — market rules, contracts, bidding markets, auctions, etc. — to coordinate players that follow their own objectives under their own private information, but have to share common resources.

In *cooperative game theory*, players can form coalitions, expecting better payoffs than by playing without cooperating.

3 Mathematical Challenges Raised by the Clean Energy Transition

As we have just seen, the clean energy transition — driven by the will to move from carbon based towards renewable energy production — is developing in a context where, on the one hand, players are more diverse in a changing energy market and, on the other hand, technology is offering more and more potential to act on the energy system at all scales. These three drivers — renewable energy push, markets expansion, technological push — are transforming the energy systems and are challenging the players in it. As a consequence, applied mathematics are sollicitated in different fields, that we sketch below. To do so, we have chosen to move from the "big picture" — energy markets with many players (see Figure 1) — down to the problem of each player. In the end, we gather transversal issues.

3.1 Game Theory, Stochastic Equilibria, Mechanism Design

- In a fully integrated vertical centralized management (when generation, distribution and supply are controlled by a unique stakeholder), optimization is indispensable to mimize costs while meeting demand. But as decision-makers are multiplying on the energy market — producers, prosumers, aggregators, cities and regions — we move from more and more from optimization towards *game theory*.

- New players come not only with their own agendas, but with their own private information — about costs of production, style of consumption, etc. — and this calls for *game theory with information*.

- The coordination of such players, to achieve supply-demand balance, can be performed through prices, as classical in economics with the notion of market equilibrium. But the stochasticity of renewable

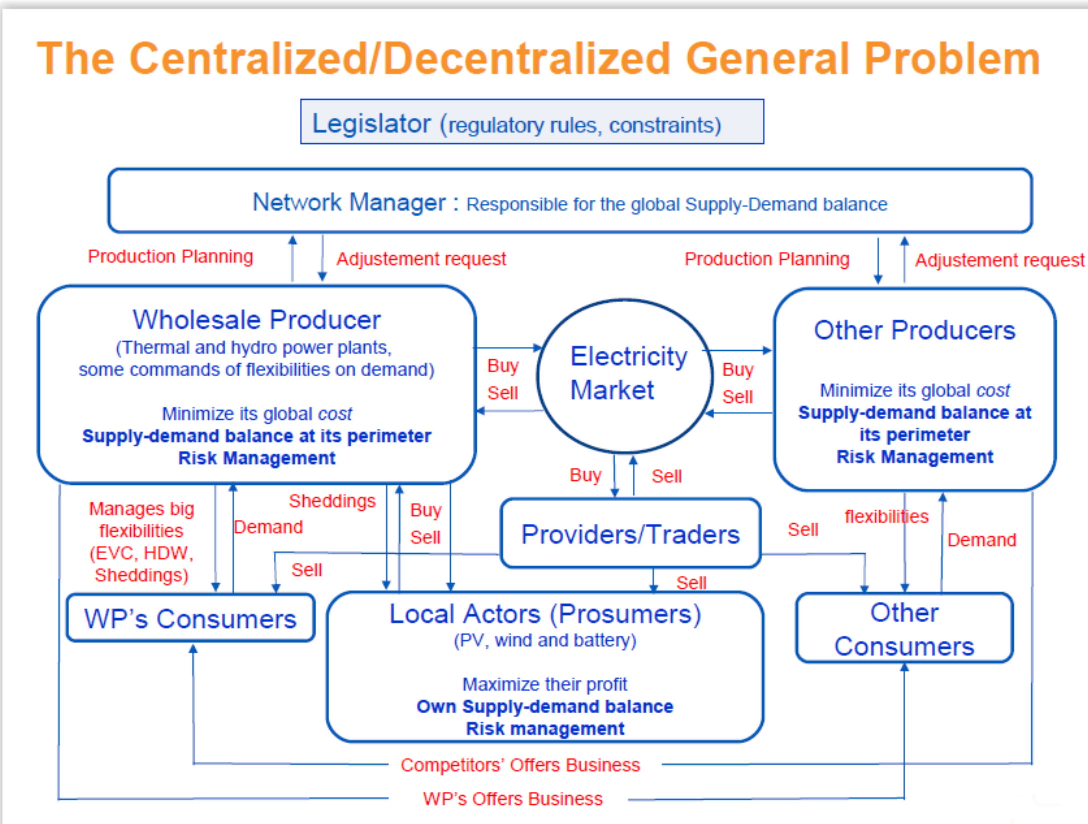

Figure 1: Overview of the centralized/decentralized general problem of energy management

energy production, and of demand, makes the analysis more delicate as we enter the realm of *stochastic equilibrium*.

- In this context, new economic regulation is called for, to achieve coordination by proper design of markets. This is the field of *mechanism design* in economics, which includes *auctions market design*, but in a challenging context: stochasticity of both supply and demand, private information of players, multiple time scales (for electricity, stability concerns make that the supply equals demand constraint has to be satisfied down to the smallest time scale, hence the existence of complex reserve markets), new economic regulation is called for.

3.2 Bilevel, decentralized and multi-stage stochastic optimization

As we move down from the coordination of players to their individual goals, we meet *optimization*.

- *Bilevel optimization* stand between game theory and optimization as it considers the interactions between a so-called principal (or many) and a so-called agent (or many). For instance, electricity suppliers propose electricity tariffs to residential customers, and the tariffs result from an optimization problem, which essentially consists in maximizing their profit. At the lower level, residential customers choose the best electricity provider for them; this choice also results from an optimization problem in which the tariffs proposed by the suppliers is an input; hence, the customers react to the decisions taken by the suppliers. Bilevel optimization problems are becoming more and more complex, because of the stochasticity of both supply and demand, of the multiplicity of principals and agents with their private information, and with multiple time scales for decision.

- *Decentralized optimization* also stand between game theory and optimization as it considers how a player can coordinate a team of decision-makers that jointly contribute to minimize a common criterion, but with decentralized information. For instance, a so-called aggregator holds both production units and devices to control customer demand (flexible electricity consumers). Thus, such a virtual power plant has its own economic objective but has to handle a team of decision-makers, the flexible consumers, that own private decentralized information. The development of smaller and smaller computer technology also pushes towards more decentralized optimization.

- The least-cost operation of production and demand control is naturally framed as a *multi-stage stochastic optimization* problem, because of the stochasticity of both supply and demand, and of the multiple time scales and steps for decision. Multiple steps come from the necessity to tackle storage (be they dams, batteries, etc.), itself a response to the need to handle the supply equals demand constraint when both sides are stochastic. Multiple time scales go from slow scale investment problems (should I invest in energy storage or develop transmission lines to the network?) to voltage and frequency regulation at very short time scale.

3.3 Transversal issues

- As we have seen, practical problems in energy management lead to large scale game, equilibrium or optimization problems. This is due to the many dimensions of the problems: scenarios of uncertainties, multiple times steps, number of decision-makers. This appeals for the development of *decomposition-coordination* based algorithms that use spatial, temporal and random structures to decompose problems into solvable subproblems, suitably coordinated.

- Another way to handle numerical complexity is to rely on convexity assumptions, tools and convexity-based algorithms. Recent approaches go *beyond convexity*, using tools from variational analysis. Handling integer variables, like in investment or capacity expansion planning problems, also forces to move beyond convexity.

- With so much variability, in time and in space, the handling of *risk* is critical, like in the issue of peak placement to avoid the random concentration of demand when stochastic supply is low. Handling risk in a dynamic setting together with mutiple decison-makers, pushes for the development of decomposition based algorithms.

- All the problems mentioned above need data as input. With the development of sensor and telecom technology, data is more and more abundant, pushing for more *statistics* and *big data analysis* (machine learning). For instance, in data-driven optimization, one can look to design tree structures that are rich enough but can be handled numerically, to provide relevant solutions.

4 Presentation Highlights

Here, we highlight the main contributions of the workshop presentations regarding the mathematical challenges raised in Sect. 3.

4.1 Introductory Session on Optimization and Energy

In an introductory talk, *Michel De Lara* broswed the new energy landscape, stressing the kind of optimization and game problems that were now more and more addressed. [36].

Jyoti Devkota put the light on the use of renewable energy in rural Nepal, mostly wood, biogas and micro hydropower. [19, 20].

4.2 Energy Markets and Strategic Planning. Equilibria

The functioning of energy markets under uncertainty leads to complex general equilibrium or game problems, which challenge numerical resolution.

Andy Philpott presented the package Julia Dynamic Generation Expansion (JuDGE) for solving stochastic capacity expansion problems, as encountered in energy planning: optimal capacity expansion in an electricity distribution network subject to reliability constraints; national capacity expansion to meet renewable energy targets; optimal transmission expansion for an electricity wholesale market with imperfectly competitive agents. The user provides JuDGE with a coarse-grained scenario tree, that models long-term uncertainties, and a JuMP formulation of a stage problem to be solved in each node. JuDGE then applies Dantzig-Wolfe decomposition to this framework; the stage problems are themselves single-stage capacity expansion problems with integer capacity variables, but quite general constraints. [57, 61, 62, 45].

Michael Ferris considered the modeling of coupled systems using an equilibrium framework, as happens in energy markets with risk where coupling comes from market constraints. He outlined standard and new mechanisms to model resulting complementarity constraints, and how to formulate the agents problems to allow risk preferences. Such problems are computationally complex and some computational results related to energy systems were given. [28, 27, 44].

Julio Deride presented a problem of strategic planning for capacity investment and production under uncertainty, in a competitive market. He modelled it as a general equilibrium problem, i. e. a collection of multi-agent optimization problems with an equilibrium constraint (supply meets demand), and proposed a solution method based on a scheme that considers: i) a stagewise-decomposition procedure using Progressive Hedging, ii) a representative agent representation, and iii) a decomposition-type method for its solution, such as the Alternating Direction Method of Multipliers (ADMM). He illustrated the numerical performance of the algorithm by solving a stochastic infrastructure planning problem for the electric vehicule fast-charging station problem over a small network. [32].

Alejandro Jofre described some of the key issues in the operational and strategic decisions when an energy system or market is stressed by a massive entry of nonconventional renewal energy production (NREP), such as the case of the Independent System Operator (ISO), the producer reactions, trade-off between cheap and uncertain generation sources and the risk analysis of producers versus generators and consumers. He used a combination of game theory, stochastic optimization and risk analysis techniques for modeling and trying to understand some of the behaviors and perturbations induced by the entry of NREP. [4, 34, 24, 25].

4.3 Hydroelectric Power Management

Hydroelectric power remains the major source of renewable energy. The management of reservoirs has long been a source of problems in multi-stage stochastic optimization.

Michel Gendreau considered the long-term energy planning of an extensive hydroelectric power system. The problem ultimately aims at evaluating the impact of additional firm load contracts on the energy reliability of the system and the future revenues for the next fifteen years, taking into account the uncertainty of future energy inflows. He proposed a method combining stochastic dynamic programming and Tabu Search approaches to solve the long-term energy-planning problem without the need to assume a prior form for the long-term persistence of future energy inflows. He compared the policies resulting from this hybrid method with policies obtained from two others versions of stochastic dynamic programming known to explicitly handle persistence of inflows: one with an additional state variable and the other coupled with a Hidden Markov Model. The results showed the effectiveness of the hybrid method in long-term persistence cases. [39, 40].

Peak/off-peak spreads in European electricity spot markets are eroding due to the nuclear phaseout and the recent growth in photovoltaic capacity. The reduced profitability of peak/off-peak arbitrage forces hydropower producers to participate in the reserve markets. *Napat Rujeerapaiboon* proposed a two-layer stochastic programming framework for the optimal operation of a multi-reservoir hydropower plant which sells energy on both the spot and the reserve markets. The backbone of this approach is a combination of decomposition and decision rule techniques. Numerical experiments demonstrate its effectiveness. [53].

4.4 Micro-Grids Stochastic Control and Dynamic Programming

Micro-grids are energy systems where decisions have to be made at many time steps, especially in and out of batteries at a short time scale. Problems are often formulated in the stochastic optimal control framework, with dynamic programming as the main method to tackle them. [8, 9, 48].

Michael Ludkovski investigated microgrid management where the controller tries to optimally dispatch a diesel generator as backup to primary renewable sources while maintaining low probability of blackouts. Dispatch takes place at discrete epochs (15 min), while balancing takes place continuously, so only probabilistic guarantees are possible. Moreover, the likelihood of a blackout during the next dispatch period is not available analytically and can only be estimated. He formulated the problem as stochastic control where the Bellman equation features local probabilistic constraints that lead to an implicit state-dependent admissible control set. To tackle this challenge, he developed novel Monte Carlo based algorithms, in particular empirical simulation procedures for learning the admissible control set as a function of system state. He proposed a variety of relevant statistical tools including logistic regression, Gaussian process regression, quantile regression and support vector machines, which he then incorporated into an overall Regression Monte Carlo (RMC) framework for approximate dynamic programming. His results indicate that using logistic or Gaussian process regression to estimate the admissibility probability outperforms the other options. His algorithms offer an efficient and reliable extension of RMC to probability-constrained control. He illustrated his findings with two case studies for the microgrid setup with time-stationary and daily-seasonal net load dynamics. [5, 38].

Michel De Lara, *Jean-Philippe Chancelier* and *Pierre Carpentier* presented three coordinated talks on decomposition-coordination methods in multi-stage stochastic optimization and how they apply to microgrid management.

Multi-stage stochastic optimization problems are, by essence, complex because their solutions are indexed both by stages (time) and by uncertainties (scenarios). Quite often, solutions are also indexed by decision units, like nodes in a graph (space), or agents in a team. Hence, their large scale nature makes decomposition methods appealing. *Michel De Lara* presented, in an unified framework, three main approaches and methods to decompose multi-stage stochastic optimization problems for numerical resolution: time decomposition (and state-based resolution methods, like Stochastic Dynamic Programming, in Stochastic Optimal Control); scenario decomposition (like Progressive Hedging in Stochastic Programming); spatial decomposition (price or resource decompositions). He outlined how writing a dynamic programming equation on the increasing sets of histories paves the way for state reduction at specified stages, making it possible to develop time block decomposition. He also showed how price or resource decompositions quite naturally provide decomposed lower and upper bounds for minimization problems. Finally, he pointed to some mathematical questions raised by the mixing (blending) of different decompositions methods to tackle large scale problems. [12].

Jean-Philippe Chancelier provided a method to decompose multi-stage stochastic optimization problems by time blocks. This method is based on reducing the so-called history space using a compressed “state” variable. It leads to a reduced dynamic programming equation. Then, he applied the reduction method by time blocks to two time-scales stochastic optimization problems arising from long-term storage management of batteries. He presented a stochastic optimization model aiming at minimizing the investment and maintenance costs of batteries for a house with solar panels. For any given capacity of battery it is necessary to compute a charge/discharge strategy as well as maintenance to maximize revenues provided by intraday energy arbitrage while ensuring a long-term aging of the storage devices. Long-term aging is a slow process while charge/discharge control of a storage handles fast dynamics. For this purpose, he showed how to design algorithms that take into account this two time scales aspect in the decision making process. He showed, on instances with several millions time steps, how one of the algorithms could tackle the optimal sizing of a storage, while taking into account charge/discharge strategy as well as aging; numerical results indicated that it could be economically significant to control aging. He also compared, on small instances, the new algorithms to the well-known Stochastic Dynamic Programming and Stochastic Dual Dynamic Programming ones: the new ones are less computationally costly, while displaying similar performances on the control of a storage.

Pierre Carpentier considered a stochastic optimization problem in which different units are connected together via a network. Each unit is a (small) control system, located at a node. Each unit state evolution is affected by uncertainties and controls of the neighboring nodes transmitted through edges. Static constraints couple all units at each time. He formulated the associated global stochastic optimization problem. He proposed two decomposition methods, whether one decouples the constraints by prices or by resources. He showed that the optimal value of the global problem can be bounded above by a sum of resource-decomposed nodal value, and below by a sum of price-decomposed nodal value. He provided conditions under which these nodal values can be computed by dynamic programming. He illustrated these results with numerical studies that tackle the optimization of urban micro-grids of large size. Finally, he introduced two different information structures for these microgrids, namely the centralized and the decentralized ones, and he analyzed the lower and upper bounds when considering these information structures. Numerical results indicate that price decomposition-based algorithms outperform the Stochastic Dual Dynamic Programming algorithm, both in terms of objective and in terms of computation running time, especially when the state dimension grows. [11, 10].

4.5 Stochastic Dual Dynamic Programming

Stochastic Dual Dynamic Programming (SDDP) is a method that partially bypasses the curse of dimensonality attached to Dynamic Programming, under convexity assumptions. It is widely used in energy management problems under uncertainty.

Multi-stage robust optimization problems, where the decision maker can dynamically react to consecutively observed realizations of the uncertain problem parameters, pose formidable theoretical and computational challenges. As a result, the existing solution approaches for this problem class typically determine suboptimal solutions under restrictive assumptions. *Angelos Georghiou* proposed a robust dual dynamic programming (RDDP) scheme for multi-stage robust optimization problems. The RDDP scheme takes advantage of the decomposable nature of these problems by bounding the costs arising in the future stages through lower and upper cost to-go functions. For problems with uncertain technology matrices and/or constraint right-hand sides, the RDDP scheme determines an optimal solution in finite time. If also the objective function and/or the recourse matrices are uncertain, the method converges asymptotically (but deterministically) to an optimal solution. The RDDP scheme does not require a relatively complete recourse, and it offers deterministic upper and lower bounds throughout the execution of the algorithm. He demonstrated the promising performance of his algorithm in a stylized inventory management problem. [30].

The Stochastic Dual Dynamic Programming (SDDP) algorithm has been used successfully for the past 25 years in the energy industry, especially for mid and long-term hydromanagement problem. In essence, SDDP is a dynamic programming algorithm that approximates the value function by outer polyhedral approximations. These outer approximations yield exact lower bounds (minimization framework), but no upper bounds. The classical way of obtaining upper bounds consists in simulating the policy over multiple scenarios and estimate the expected cost. This approach has multiple drawbacks. First, the bound is only estimated, hence using it as a stopping test can lead to false positives; the theoretical probability of false positive can become very high if it is often tested. A second way of obtaining upper bounds consists in leveraging the monotonicity of Bellman Operators and the convexity of the value function. More precisely, from an upper bound at time $t + 1$ one can compute upper bound at time t on a finite set of points. Convexity allows to extend the upper bound definition on the convex hull of this set. Finally, *Vincent Leclère* presented a third approach consisting in applying SDDP to the Fenchel transform of the value function. The exact lower bound of the Fenchel transform becomes a deterministic upper bound on the original value function. This approach leads to exact upper bounds converging toward the true value. Incidentally, this also defines a new policy with guaranteed expected cost. [37].

Regan Baucke discussed how recent advances in SDDP methods for finite horizon problems can be transfered to the infinite horizon setting. These types of problems arise when modelling energy systems over long time horizons. He proposed a convergent algorithm with several attractive properties; chiefly no Monte Carlo simulation is required to obtain an upper bound. [6, 7].

Bernardo Freitas Paulo da Costa presented a new algorithm for solving multi-stage stochastic mixed integer linear programming (MILP) problems with complete continuous recourse. A typical example of such problems is the energy planning with disjunctive operational constraints. Similar to cutting plane methods, he introduced nonlinear Lipschitz cuts that are building blocks for lower approximations for nonconvex cost-to-go functions. An example of such cuts are those derived from (exact) Augmented Lagrangian Duality for MILPs. If one chooses a family of Lipschitz cuts that is MILP representable, the introduction of these cuts does not change the class of the original stochastic optimization problem. He illustrated the application of this algorithm, comparing his approach with the convex relaxation of the stagewise problem, for which one can apply SDDP, and for a discretized approximation, applying SDDiP (where "i" stands for integer SDDP). [1].

Alexander Shapiro discussed computational approaches to solving convex stochastic programming problems. He started with a discussion of sample complexity of solving static problems and argued that this is essentially different from sample complexity of solving multi-stage programs. In some applications the considered multi-stage stochastic programs have a periodical behavior. He demonstrated that in such cases it is possible to drastically reduce the number of stages by introducing a periodical analog of the so-called Bellman equations, used in Markov Decision Processes and Stochastic Optimal Control. Furthermore, he described a variant of the Stochastic Dual Dynamic Programming algorithm, applied to the constructed periodical Bellman equations, and showed numerical experiments for the Brazilian interconnected power system problem. [55, 21].

4.6 Stochastic Programming

Stochastic Programming, where decisions are indexed by a scenario tree (capturing the sequential flow of information), is one of the most popular approach to tackle multi-stage stochastic optimization. It is widely used in energy management problems under uncertainty. [54, 43].

The progressive hedging algorithm minimizes an expected "cost" by iteratively decomposing into separate subproblems for each scenario. Up to now, it has depended on convexity of the underlying "cost" function with respect to the decision variables and the constraints on them. However, *Terry Rockafellar* presented a new advance that makes it possible to obtain convergence to a locally optimal solution when the procedure is executed close enough to it and a kind of second-order local sufficiency condition is satisfied. Besides applications in which costs and associated constraints may directly be nonconvex, there are applications to stochastic programming problems in which those are convex but the probabilities for the scenarios may be decision-dependent. For example, in a two-stage problem the probabilities in the recourse stage could be influenced by the first-stage decision. [49, 50].

Thanks to linear programming duality, the classical linear stochastic program with recourse obeys convexity and duality properties facilitating the development of theory and the design of algorithms considerably. Practical needs in operations research and elsewhere quickly drove the modeling beyond the convex case. For instance, 0-1 decisions and related integer variables to model switching and indivisibility or nonlinearites of physical nature, such as squared-differences of square expressions marking drop of potential related to flow along pipes. *Ruediger Schultz* put into perspective extensions of the classical model concerning all key ingredients, namely the measure, the integrand, the second-stage optimization problem, and the nonanticipative first-stage variable. Since convexity is lost instantly when extending the model, identification of proper mathematical alternatives is becoming crucial. A promising source for such alternatives is provided by computer algebra which opens up a rich world of approaches in mathematical structures not having been in the research focus until very recently. Foremostly, it is Ideal Theory that has been addressed in this respect. Some first successful approaches using algebraically motivated structures and methods were presented. [2, 33].

Claudia Sagastizábal proposed a convergent primal-dual solution algorithm for nonconvex optimization problems with nonlinear constraints, possibly nonsmooth. The approach applies a proximal bundle method to a dual problem that arises in the context of generalized augmented Lagrangians and that yields zero duality gap. The methodology is tailored so that Lagrangian subproblems can be solved inexactly without hindering the primal-dual convergence properties of the algorithm. Primal convergence is ensured even when the dual solution set is empty. [52, 14, 29].

Traditional stochastic programs optimize the expected value of some function that depends on the decision variables as well as on some random variables that represent the uncertainty in the problem. Such formulations assume that the probability distribution of those random variables is known. However, in practice the probability distribution oftentimes is not known or cannot be accurately approximated. One way to address such ambiguity is to work with distributionally robust stochastic programs (DRSP), which minimize the worst-case expected value with respect to a set of probability distributions. *Tito Homem-de-Mello* discussed some recent advances in the research on DRSPs. In particular, he studied the question of how to identify the critical scenarios obtained after solving a multi-stage DRSP. Recent research has studied the notion of effective scenarios for static distributionally robust stochastic programs. Roughly speaking, a scenario is deemed effective if its removal changes the optimal value of the problem. We discuss the extension of these ideas to the case of multi-stage stochastic programs. Such extension requires proper ways of defining the meaning of removing a scenario in a dynamic context. Computational and analytical results show that identifying such effective scenarios may provide useful insight on the underlying uncertainties of the problem. [47, 46].

4.7 Risk, Robustness and Partially Observed Systems

Darinka Dentcheva analyzed composite functionals representing distributional characteristics of random data. The functionals depend on the decision maker's choice when used as objectives in optimization problems. Although, frequently, models of risk are nonlinear with respect to the underlying distributions, they often can be represented as structured compositions. She considered the use of smooth estimators with particular attention being paid to kernel estimators for composite functionals and for the optimal value of optimization problems using those as objectives. Strong law of large numbers for the estimators, for the optimal values and the optimal solutions are established under mild conditions on the functions involved. Central limit theorems for the estimated composite functionals and the optimal value of composite optimization problems are presented as well. She compared the performance of the estimators to the empirical estimators numerically, and discussed several popular risk measures as illustrative examples. While many known coherent measures of risk can be cast in the presented structures, she emphasized that the results are of more general nature with a wider applicability. Applications of the results to hypothesis testing of stochastic orders and portfolio efficiency were outlined. [15, 18, 17].

Andrzej Ruszczyński introduced the concept of a risk form, which is a real functional on the product of two spaces: the space of measurable functions and the space of measures on a Polish space. He presented a dual representation of risk forms and generalized the classical Kusuoka representation to this setting. For a risk form acting on a product space, he defined marginal and conditional forms and proved a disintegration formula, which represents a risk form as a composition of its marginal and conditional forms. He applied the proposed approach to two-stage optimization problems with partial information and decision-dependent observation distribution. Next, he considered risk measurement in controlled partially observable Markov systems in discrete time. In such systems, part of the state vector is not observed, but affects the transition kernel and the costs. He introduced new concepts of risk filters and studied their properties. He also introduced the concept of conditional stochastic time consistency. He derived the structure of risk filters enjoying this property and proved that they can be represented by a collection of law invariant risk measures on the space of functions of the observable part of the state. He also derived the corresponding dynamic programming equations. [16, 15, 26].

It is well known that Sample Average Approximations (or Empirical Risk Minimization) can lead to arbitrarily poor solutions and slow learning when the objective function is poorly behaved. *Johannes Royset* described a surprisingly simple remedy that he coined Diametrical Stochastic Optimization (Diametrical Risk Minimization). In contrast to common robustification strategies based on perturbing the data set and probability distribution, his approach "diametrically" modifies any solution and thereby obtains theoretical stability guarantees even for poorly behaved functions. He showed that, in challenging machine learning problems, the approach generalizes even if obtained after aggressive minimization of the diametrical risk. [42].

The standard approach for modeling partially observed systems is to model them as partially observable Markov decision processes (POMDPs) and obtain a dynamic program in terms of a belief state. The belief state formulation works well for planning, but is not ideal for online reinforcement learning because the belief state depends on the model and, as such, is not observable when the model is unknown. *Aditya Mahajan* presented an alternative notion of an information state for obtaining a dynamic program in partially observed models. In particular, an information state is a sufficient statistic for the current reward which evolves in a controlled Markov manner. He showed that such an information state leads to a dynamic programming decomposition. Then he presented a notion of an approximate information state, and an approximate dynamic program based on the approximate information state. Approximate information state is defined in terms of properties that can be estimated using sampled trajectories. Therefore, they provide a constructive method for reinforcement learning in partially observed systems. [59].

4.8 Pitch Session of PhD students

Two PhD students briefly presented their current work. *Thomas Martin* introduced the procurement problem — or how a company acquires the ressources needed for its activity — and presented a series of mathematical formulations incorporating stochasticity and dynamics. *Cyrille Vessaire* sketched the problem of optimization of reservoir development and design under uncertainty.

5 Scientific Progress Made

Here, among the many talks, we highlight themes that were tackled by several participants and that lead to substantial contributions.

5.1 Algorithms and softwares for stochastic equilibrium

One of the big issue for computing Nash equilibrium in the setting of electricity markets comes from the fact that, in several cases, the agent payoffs are nonconvex and sometimes even discontinuous. These facts imply that, generally, we are looking for mixed strategies, that is, multilinear payoff functions; but, then, some local stability of algorithms — as a result of strong concavities or quadratic approximations — are not available. The other interesting point is that the strategy sets for the players is contained in an infinite dimensional space, which requires good discretization and/or approximation schemes. All these complexities make possible to calculate Nash equilibria for a few number of agents; however these constrains are not so restrictive because we are in an oligopolistic competition setting.

5.2 Decomposition methods

As we have seen, practical problems in energy management are large scale and, thus, possibly amenable to decomposition methods. At the workshop, several presentations addressed these issues and discussed the advances of the theory and methods in this area.

Large-scale multi-stage stochastic optimization problems are complex because their solutions are indexed by stages (time) by scenarios (uncertainty) and by units (space). Since long, decomposition methods exist to tackle each one of these three dimensions. New mathematical developments and new computational methods are now needed to be able to, on the one hand, mix several kinds of decomposition on the same problem and, on the other hand, to handle nonconvex and nonsmooth problems.

These points were all addressed during the conference, and their simultaneous use is the next step. In particular, mixing nonconvex progressive hedging method with time block dynamic programming method (or more generally with price and resource decomposition methods) now seems within reach.

5.3 Stochastic Dual Dynamic Programming

One theoretical and practical issue with the stochastic dual dynamic programming method is the question of deciding when a particular problem has been "solved". Traditionally, convergence has been determined by an expensive statistical test, which can be prone to false positives. This has been the de-facto approach since the method's inception. Several talks at this conference focused on the development of better convergence criteria for the stochastic dual programming method. There is a growing consensus in the community that these convergence tests are a remedy to many questions and concerns about the performance of the method, especially from those who use the method in practice. These new convergence tests result in more consistent and robust solutions, with which stronger theoretical guarantees on their performance can be made. Moreover, the workshop provided the opportunity for multiple discussions on extension of the methodology to the infinite horizon framework using various approaches.

5.4 Risk

Evaluation and optimization of risk is one of the central issues in the analysis of energy systems. Demand and renewable supply are subject to high uncertainty, with some scenarios associated with very large or even catastrophic costs. Modeling and control of risk require new mathematical and statistical theories and new computational methods. The difficulties are compounded in the presence of incomplete information, when some essential characteristics of the system cannot be directly observed. The combination of all these challenges in a dynamical system requires, therefore, risk-averse statistical filtering and learning, and risk-averse dynamic control. At the workshop, several presentations addressed these issues and discussed the advances of the theory and methods in this area. In particular, with the introduction of risk forms, we now have a rich framework to handle risk in optimization problems.

6 Outcome of the Meeting

One outcome of the meeting was the discovery, by some participants, of the extent to which the transformation of the energy sector triggered research in optimization and game theory.

At the end of the meeting, we organized a wrap-up session. We asked the participants which were the main themes they saw relevant to tackle at the issue of the workshop. Here is the list: decomposition, decentralization; complexity, algorithms, softwares; data-driven, machine learning; convexity and beyond; risk.

Was also discussed the opportunity to investigate the field of decision-making in climate change mitigation and adaptation. Indeed, the Intergovernmental Panel on Climate Change (IPCC) displays climate evolution scenarios, as well as economic ones, that could serve as inputs for optimization and games problems.

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