Even numbered problems

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Parovicenko need not have these properties; per (3) there can even be a rigid Parovicenko space

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What are the absolute retracts of $\mathbb{N}^*$? Szymanski: CH characterization. (Simon: not all compact separable subspaces using indep matrices)

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this can lead us to (Boban's) tie-points

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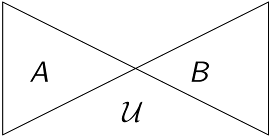

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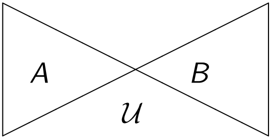

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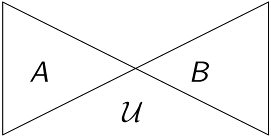

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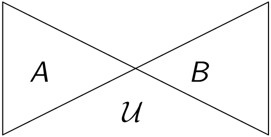

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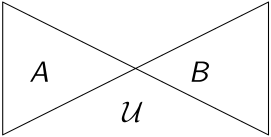

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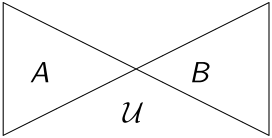

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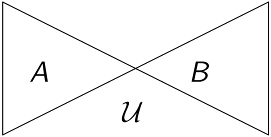

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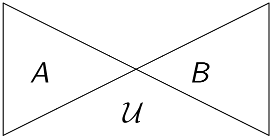

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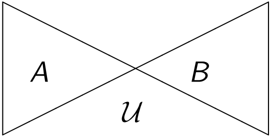

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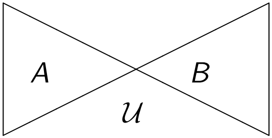

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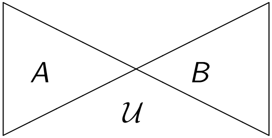

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4. similar to: is every point of $\mathbb{N}^*$ a butterfly point? MA$\models$ yes
   Is $\mathbb{N}^* \setminus \{\mathcal{U}\}$ ever normal?

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$\mathfrak{b} = \mathfrak{c}$ implies Yes, but I haven't seen any other constructions.

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is there compact sequential order more than 2?

If ω sits in compact sequential X, then there is a madf $\mathcal{A}$ on ω consisting of converging sequences. If these are all distinct points, then this is an interesting madf. [partition algebras]

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T-algebras also involve tie-points (but not of $\mathbb{N}^*$)

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a sequence $\{a_\alpha : \alpha \in \gamma\} \subset \mathcal{P}(\mathbb{N})$ coherently minimally generates B if for all $\alpha < \gamma$, $\{a_\beta \wedge a_\alpha : \beta < \alpha\}$ generates the factor $B[a_\alpha]$.

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A family $\{a_t : t \in \text{Succ}(T)\}$ is a T-algebra if $T \subset 2^{<\mathfrak{c}}$ is such that no element has a unique immediate successor, for all $t^\frown 0 \in T$, $\{a_{t^\frown 0}, a_{t^\frown 1}\}$ are complements and for all branches ρ of T (not just maximal) $\{a_{\rho\restriction\alpha+1} : \rho \restriction \alpha{+}1 \in T\}$ is a coherent minimal generating sequence.

Can there be a T-algebra such that ...?

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Theorem (with K.P. Hart)

If there is a Mahlo cardinal then there is a forcing extension in which Moore-Mrowka holds and with a T-algebra (and $T = 2^{<\omega_1}$) that gives compact sequential with no points of countable character.

more T-algebras needed

Hušek

Can there be a compact ω-free and ω_1-free space?

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Question (Hušek, Juhasz)

Does every compact space of countable tightness have a point of character at most $\aleph_1$?

what aren't T-algebras good for?

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Definition (updated)

A compact X has a small diagonal if X^2/Δ_X is ω_1-free.

Original: if $\{\{x_\alpha, y_\alpha\} : \alpha \in \omega_1\} \subset [X]^2$, there is an open F_σ's splitting $\aleph_1$-many of the pairs.

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6. can the space be "mostly metrizable"?

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Have X as a subspace of 2^{ω_2}. for each $x \in X$ and $\alpha \in \omega_2$, let $[x \restriction \alpha]$ be the usual closed subset of X.

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Question

Could there be an example where the order-type of each L_x is some ω^n? (or bounded above in ω^ω).

Conjecture: ccc Souslin free iteration (splitting ω_1 sequences like producing Q-sets in $[0, 1]$.

Question

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Discussion

Let Y be a sequentially compact space of compact tightness, perhaps $h\pi\chi(Y) = \omega$. Construct / find / postulate a maximal free filter $\mathcal{F}$ of closed subsets of Y.

Define proper poset $\mathbb{P}$ by $p : \mathcal{M}_p \to Y$ according to $M_1 \in M_2 \in \mathcal{M}_p$ implies $p(M_1) \in M_2 \cap \bigcap\{\overline{F \cap M_1} : F \in \mathcal{F} \cap M_1\}$. Possibly more conditions on the choice of $p(M)$. e.g. $\chi = \omega$

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Does PFA imply that Y contains a copy of ω_1? or not

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Does PFA(S) imply that if Y has a countably tight compactification, then we have, or can S-preserving force, an S-indestructible maximal filter $\mathcal{F}$? Conclude that having Souslin S does not imply there is a Moore-Mrowka space.

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