Approximation Algorithms for the Traveling Repairman Problem on a Line

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Traveling Repairman Problem

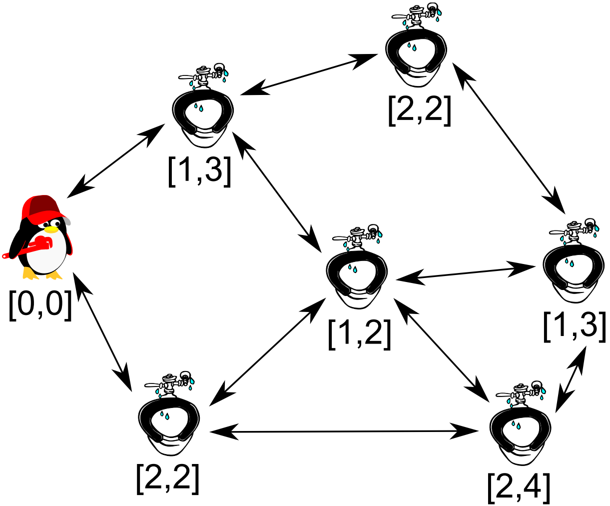
A repairman gets service requests, each with a location and a valid time window. The repairman wants to maximize the amount of attended service requests, subject to move at a certain maximum speed.

Traveling Repairman Problem

Traveling Repairman Problem (with networks)

Let $G = (V, E)$ be a complete graph and $r \in V$. Each edge $e \in E$ has length $\ell_e > 0$ and each vertex $v \in V$ has a time window $[a_v, b_v]$. The repairman starts on vertex r at time $t = 0$ and can move along the edges of G at *unit* maximum speed. Find a route for the repairman that starts at r and maximizes the amount of vertices visited during their time window.

Example



Complexity

Hamiltonian Path

Let $H = (V, F)$ be an arbitrary graph. Consider the instance of the traveling repairman problem given by $G = (V, E)$, $\ell_e = 1$ if $e \in F$ and $\ell_e = |V|$ if $e \notin F$, and $a_v = 0$, $b_v = |V| - 1$ for all $v \in V$. Then:

- If H has a Hamiltonian path starting at $r \in V$ then the repairman can visit all vertices during their time window.
- If H does not have a Hamiltonian path then the repairman cannot visit all vertices during their time window.

Metric lengths

Metric

We say that the lengths ℓ form a metric if:

- $\ell_{uv} = \ell_{vu} \geq 0$ for all u, v,
- $\ell_{uv} = 0$ if and only if $u = v$,
- $\ell_{uw} + \ell_{wv} \geq \ell_{uw}$ for all u, v, w.

Examples

- Euclidean.
- Manhattan.
- Graph (general, tree, path).

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Unit time windows

Tree metric

Let $T = (V, E)$ be a tree and $\ell_e > 0$ for $e \in E$. Let G be a complete graph on V and extend ℓ in such a way that ℓ_{uv} is the length of the path between u and v in T.

Complexity

The Traveling Repairman Problem with unit time windows is NP-hard even with a tree metric[a].

[a] Frederickson and Wittman. *Approximation algorithms for the traveling repairman and speeding deliveryman problems*. Algorithmica (2012) 62:1198-1221.

Problem on a line

Line metric

Let $P = (V, E)$ be a path and $\ell_e > 0$ for $e \in E$. Let G be a complete graph on V and extend ℓ in such a way that ℓ_{uv} is the length of the path between u and v in P.

Complexity

The Traveling Repairman Problem with arbitrary time windows is NP-hard even with a line metric[a].

[a] Tsitsiklis. *Special cases of traveling salesman and repairman problems with time windows*. Networks (1992) 22:263-282.

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Unit time windows on a line

Complexity

Unknown: open problem since 1992.

Approximation

- 2005 Guarantee 8 and time $O(n^2)^a$.
- 2005 Guarantee $4+\epsilon$ and time $O(n^{8/\epsilon})^a$.
- 2012 Guarantee 3 and time $O(n^4)$ (Frederickson, Wittman).
- 2013 Guarantee 4 and time $O(n^2)$ (López, Pérez, Urbán, Z.).
- 2014 Guarantee 3 and time $O(n^2)$ (López, R. , Urbán, Z.).
- 2015 Tight analysis (R. and Z.).

[a] Bar-Yehuda, Even, and Shahar. *On approximating a geometric prize-collecting traveling salesman problem with time windows*. Journal of Algorithms (2005) 55:76-92

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Approximation Algorithm

Sketch of the algorithm

Rotate Rotate the input 45 degrees.

Grid Add a *unit* grid.

Delete Delete unused rows and columns.

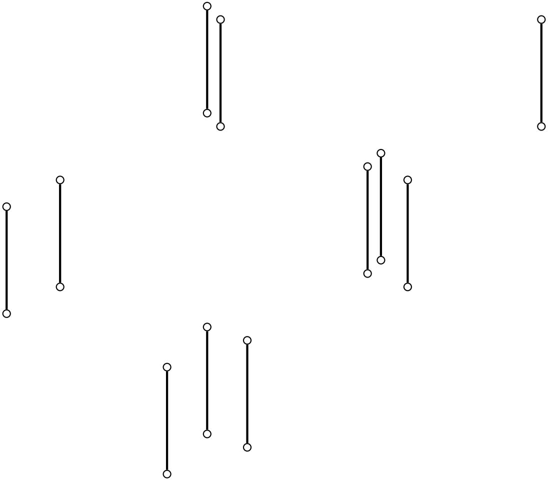
Costs Assign costs to the grid.

Route Find an optimal route on the grid.

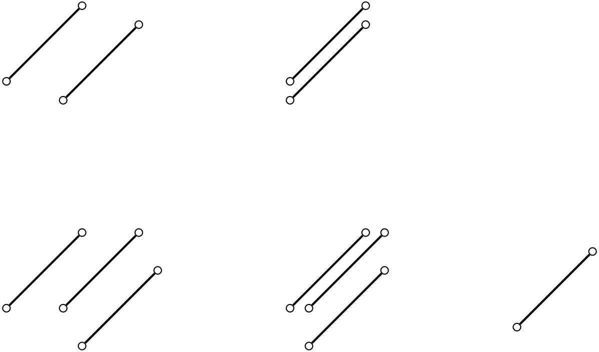
Improvement

We use an improved DAG that avoids double counting due to Pérez, Urbán, López, and Z (2014).

Example



Example



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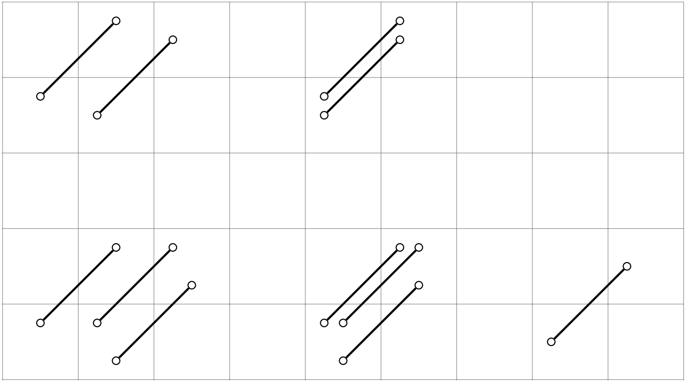
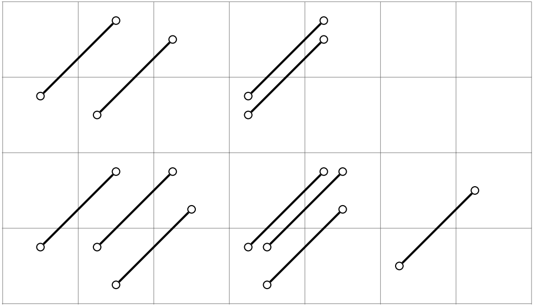


Example



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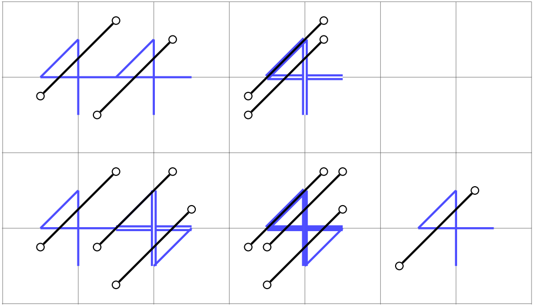

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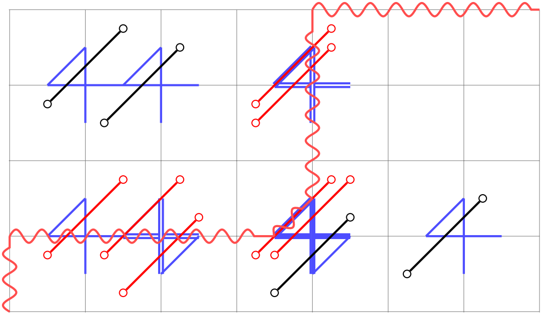

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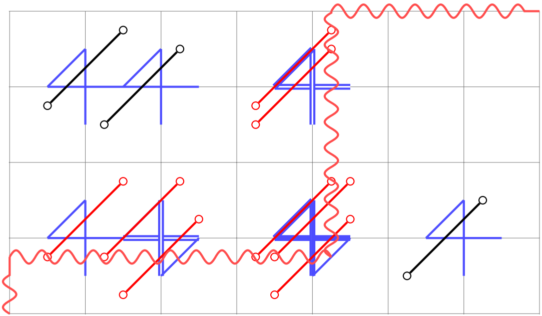

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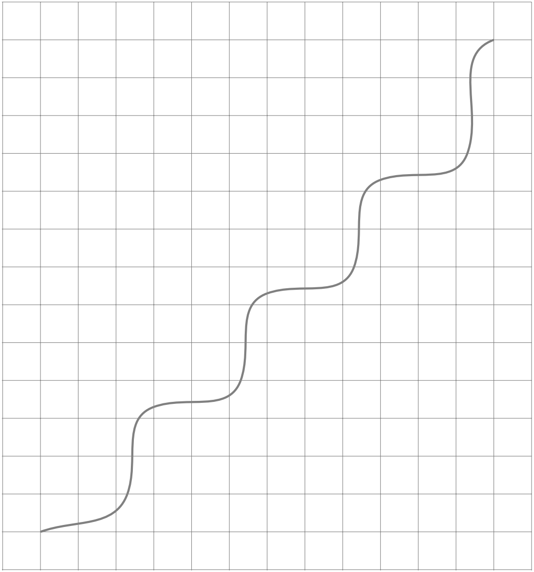
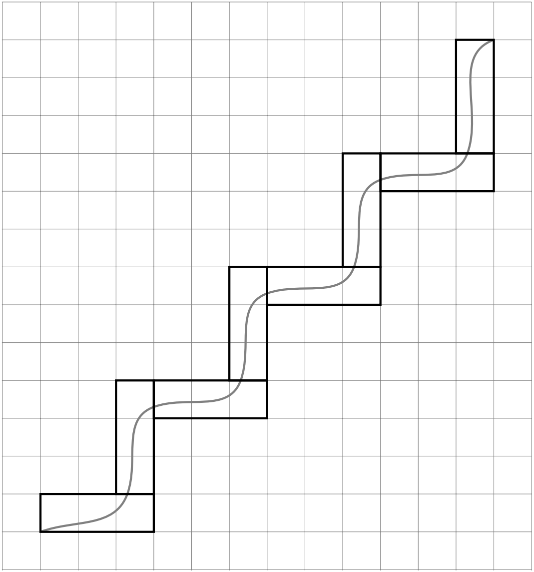
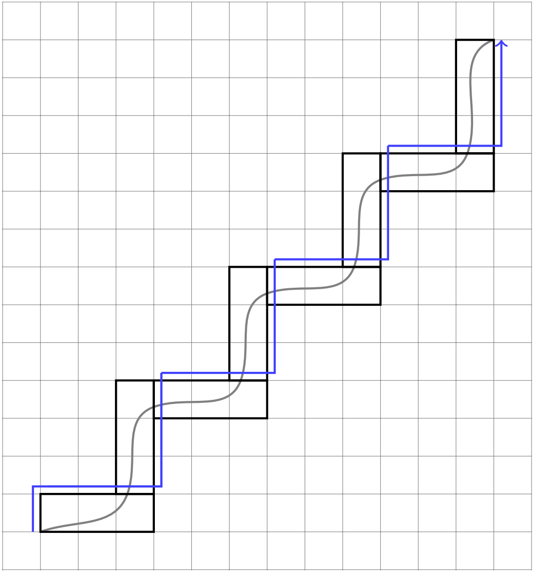
Approximation guarantee 4



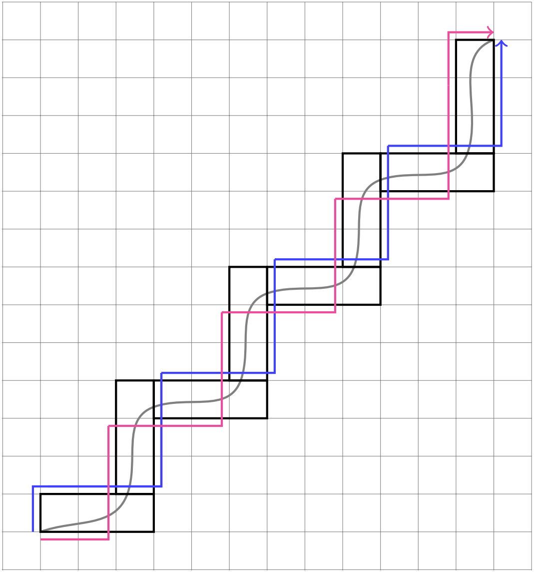
Approximation guarantee 4



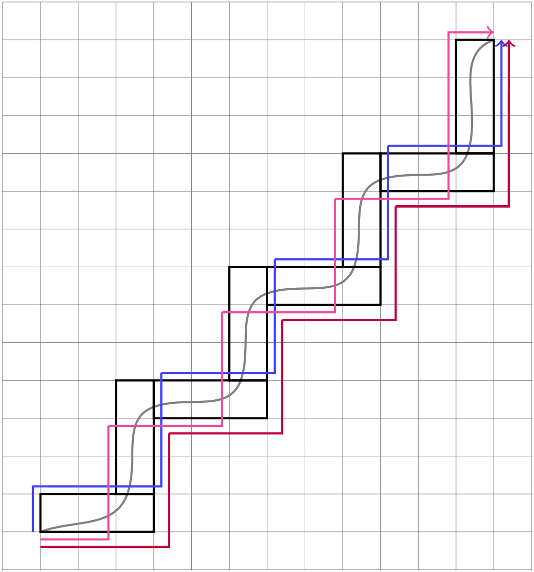
Approximation guarantee 4



Approximation guarantee 4



Approximation guarantee 4



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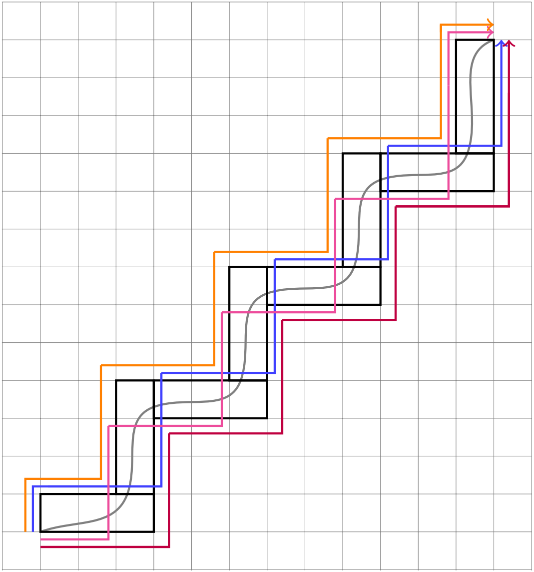
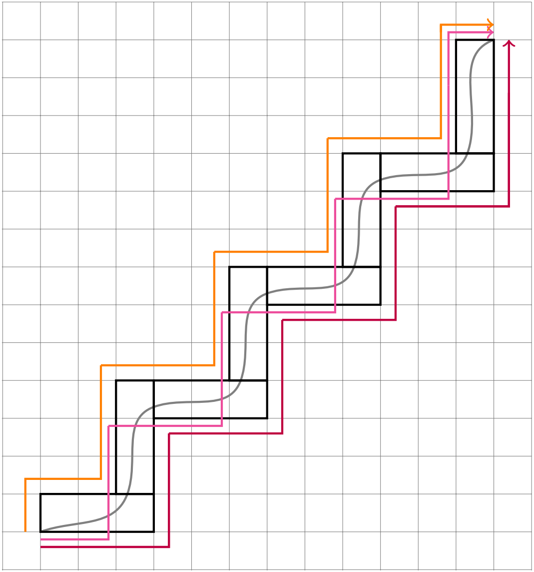


Approximation guarantee 3



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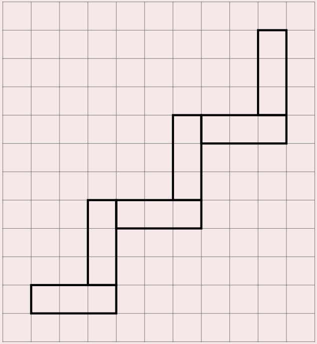
Tight analysis

Periodic routes

Note that in the previous two analysis we used a few periodic routes to find upper bounds on the approximation guarantee. We wanted to know if this approach would lead to a better analysis.

A ladder with 2 steps



Linear programming

It is possible to set up a linear program as follows:

- Consider a ladder with k steps.
- For each type t of intersection between input segments and the ladder, let x_t be the fraction of segments of this type among all.
- For each grid path p between the origin and the end point of the ladder, consider the sum s_p of all x_t such that t is intersected by p.
- Solve the linear program $\min z$ subject to $s_p \leq z$ and $\sum x_t = 1$.

Solving the linear program

Optimal solutions

k	constraints	z^*	guarantee
1	21	0.3333	3.0000
2	121	0.3529	2.8333
3	705	0.3478	2.8750
4	4109	0.3448	2.9000
5	23949	0.3428	2.9167
6	139585	0.3415	2.9286
7	813561	0.3404	2.9375

Solving the linear program

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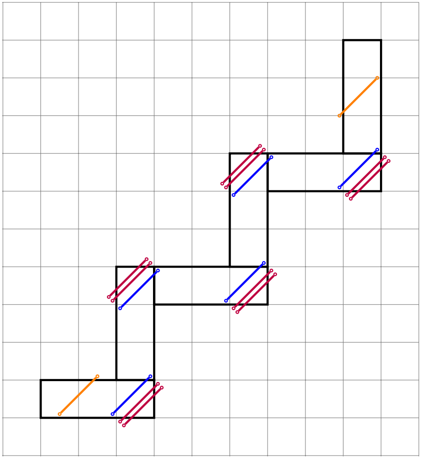
This looks like

$$\lim_{k \to \infty} \frac{6k+5}{2k+2} \to 3$$

A tight family of instances



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Future work

Open questions

- What is the complexity of the problem?
- What is the best guarantee achievable in quadratic time?
- Is there a PTAS or is there a bound on approximability?