

Polynomials: Counting and Stability

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1 Overview of the workshop topics

Understanding the real solutions of polynomials or systems of polynomial equations is at the heart of many problems both in the realm of pure mathematics and in the mathematical modeling of networks in systems biology, in computer aided geometric design, or in the control of mechanisms. While there exist a lot of celebrated results (for example, bounds exploiting the signs of the coefficients of a univariate polynomial or bounds taking into account the number of terms), even some basic questions are still unsolved.

There has been a recent explosion of activity in this area, where different tools have started to be connected in new and unexpected ways, such as tools from the theory of matroids, from fewnomial theory, or from convex algebraic geometry. This virtual workshop has brought together experts and young researchers from from close but different areas in real algebraic geometry to collaborate and to lead to further progress (both theoretical and practical) on a number of problems on the real solutions of polynomials. Specific consideration was given on the problem of counting and bounding the number of real roots as well as on questions of stability.

2 Process of the workshop and presentation highlights

The workshop took place online, as a consequence of the global pandemic situation. The format made it possible to allow participation of a higher number of people than for a usual workshop at Casa Matemática Oaxaca. 81 participants from very different time zones in the world (such as Central Europe, the West Coast of the United States and Malaysia) attended the virtual event, among them also early-career researchers. Altogether, the workshop featured 22 virtual talks. To accommodate the participants in various time, the organizers decided to go for a dense format, but still allowing ample time for questions and discussions.

The spectrum of the talks spanned from classical algebraic geometry to recent developments in real convex algebraic geometry, geometry of polynomials and polynomial optimization. On the first day, three panoramic talks introducing to the topics of the workshops were given, partly by the organizers. The workshop also included short talks by early career researchers. In detail, the following talks were given.

- Determinantal representations and the image of the principal minor map (Abeer Al Ahmadieh)
- Fewnomial bounds and multivariate generalisations of Descartes' rule of signs (Frédéric Bihan)

- Stable polynomials and related families of polynomials (Petter Brändén)
- Real degeneracy loci of matrices and hyperbolicity cones of real polynomials (Papri Dey)
- Real zeros of sums of nonnegative circuit polynomials (Mareike Dressler)
- A polyhedral description for the non-properness of a polynomial map (Boulous El Hilany)
- Inflection polynomials of linear series on superelliptic curves (Cristhian Garay López)
- The many faces of polynomial capacity (Khazhgali Kozhasov)
- Matroids with the half-plane property and related concepts (Mario Kummer)
- Hyperbolic plane curves near the non-singular tropical limit (Cédric Le Texier)
- On maximally inflected hyperbolic curves (Lucía López de Medrano)
- Spectrahedral representations of hyperbolic plane curves (Simone Naldi)
- Counting pieces of real near-circuit hypersurfaces faster (J. Maurice Rojas)
- Imaginary projections: complex versus real coefficients (Mahsa Sayyary Namin)
- Low-complexity semidefinite representation of convex hulls of curves (Claus Scheiderer)
- Return of the plane evolute (Boris Shapiro)
- On generalizing Descartes' rule of signs to hypersurfaces (Máté László Telek)
- Conic stability of polynomials, imaginary projections and spectrahedra (Thorsten Theobald)
- Metric restrictions on the number of real zeros (Josué Tonelli-Cueto)
- Harmonic hierarchies for polynomial optimization (Mauricio Velasco)
- Log-concavity and applications to approximate counting and sampling in matroids (Cynthia Vinzant)
- Positively hyperbolic varieties, tropicalization and positroids (Josephine Yu)

During all the days, the virtual talks were lively, accompanied by many questions and comments of the participants. The workshop also featured a session on open problems and future directions.

3 Outcome of the Meeting

The organizers were very happy that, despite the circumstances, the meeting served as an important and inspiring gathering of researchers in the various branches of real polynomials. The discussions in the public virtual sessions raised important questions and – to the extent which is possible – initiated new research projects and collaborations. Specifically, the talks and the discussions revealed and fostered many timely connections between real polynomials and convexity, combinatorics, computational aspects, polyhedral geometry, semialgebraic geometry and tropical geometry. The workshop also identified specific open problems and research directions around the connections of fewnomials and stability, the computation of bounds for real roots via oriented matroid theory, the possible supports of square polynomial systems all whose complex roots are real, on the use of modern tools to explore multivariate generalizations of Voorhoeve's theorem from 1976, etc.